Lesson 5: Negative Rational Exponents

- Let's investigate negative exponents.

5.1: Math Talk: Don't Be Negative

Evaluate mentally.

9^2 9^{-2} $9^{\frac{1}{2}}$ $9^{-\frac{1}{2}}$

5.2: Negative Fractional Powers Are Just Numbers

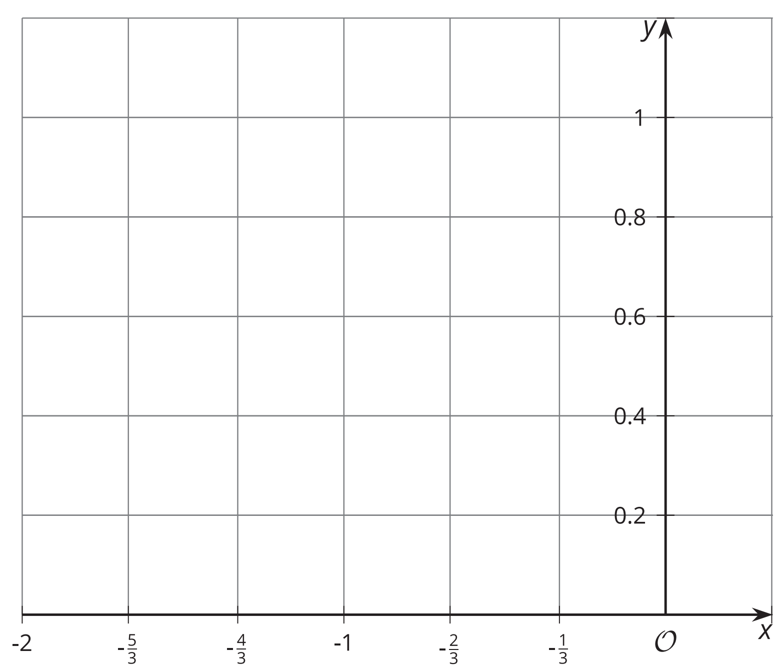
1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

x	-2	$-\frac{5}{3}$	$-\frac{4}{3}$	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0
2^x (using exponents)	2^{-2}	$2^{-\frac{5}{3}}$	$2^{-\frac{4}{3}}$	2^{-1}	$2^{-\frac{2}{3}}$	$2^{-\frac{1}{3}}$	2^0
2^x (decimal approximation)							

a. Plot these powers of 2 in the coordinate plane.

b. Connect the points as smoothly as you can.

c. Use your graph of $y = 2^x$ to estimate the value of the other powers in the table, and write your estimates in the table.



2. Let's investigate $2^{-\frac{1}{3}}$.

a. Write $2^{-\frac{1}{3}}$ using radical notation.

b. What is the value of $\left(2^{-\frac{1}{3}}\right)^3$?

c. Raise your estimate of $2^{-\frac{1}{3}}$ to the third power. What should it be? How close did you get?

3. Let's investigate $2^{-\frac{2}{3}}$.

a. Write $2^{-\frac{2}{3}}$ using radical notation.

b. What is $\left(2^{-\frac{2}{3}}\right)^3$?

c. Raise your estimate of $2^{-\frac{2}{3}}$ to the third power. What should it be? How close did you get?

5.3: Any Fraction Can Be an Exponent

1. For each set of 3 numbers, cross out the expression that is not equal to the other two expressions.
   a. $8^{\frac{4}{5}}$, $\sqrt[4]{8}^5$, $\sqrt[5]{8}^4$

   b. $8^{-\frac{4}{5}}$, $\frac{1}{\sqrt[5]{8^4}}$, $-\frac{1}{\sqrt[5]{8^4}}$

   c. $\sqrt{4^3}$, $4^{\frac{3}{2}}$, $4^{\frac{2}{3}}$

   d. $\frac{1}{\sqrt{4^3}}$, $-4^{\frac{3}{2}}$, $4^{-\frac{3}{2}}$

2. For each expression, write an equivalent expression using radicals.
   a. $17^{\frac{3}{2}}$

   b. $31^{-\frac{3}{2}}$

3. For each expression, write an equivalent expression using only exponents.
   a. $\left(\sqrt{3}\right)^4$

   b. $\frac{1}{\left(\sqrt[3]{5}\right)^6}$

Are you ready for more?

Write two different expressions that involve only roots and powers of 2 which are equivalent to $\frac{4^{\frac{2}{3}}}{8^{\frac{1}{4}}}$.

5.4: Make These Exponents Less Complicated

Match expressions into groups according to whether they are equal. Be prepared to explain your reasoning.

$\left(\sqrt{3}\right)^4$

$\sqrt{3^2}$

$\left(3^{\frac{1}{2}}\right)^4$

$(\sqrt{3})^2 \cdot (\sqrt{3})^2$

$\left(3^2\right)^{\frac{1}{2}}$

3^2

$3^{\frac{4}{2}}$

$\left(3^{\frac{1}{2}}\right)^2$

Lesson 5 Summary

When we have a number with a negative exponent, it just means we need to find the reciprocal of the number with the exponent that has the same magnitude, but is positive. Here are two examples:

$$7^{-5} = \frac{1}{7^5}$$

$$7^{-\frac{6}{5}} = \frac{1}{7^{\frac{6}{5}}}$$

The table shows a few more examples of exponents that are fractions and their radical equivalents.

x	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1
5^x (using exponents)	5^{-1}	$5^{-\frac{2}{3}}$	$5^{-\frac{1}{3}}$	5^0	$5^{\frac{1}{3}}$	$5^{\frac{2}{3}}$	5^1
5^x (equivalent expressions)	$\frac{1}{5}$	$\frac{1}{\sqrt[3]{5^2}}$ or $\frac{1}{\sqrt[3]{25}}$	$\frac{1}{\sqrt[3]{5}}$	1	$\sqrt[3]{5}$	$\sqrt[3]{5^2}$ or $\sqrt[3]{25}$	5

Lesson 5 Practice Problems

1. Write each expression in the form a^b, without using any radicals.

   a. $\sqrt{5^9}$

   b. $\frac{1}{\sqrt[3]{12}}$

2. Write $32^{-\frac{2}{5}}$ without using exponents or radicals.

3. Match the equivalent expressions.

   A. $8^{\frac{1}{3}}$

   B. $8^{-\frac{1}{3}}$

   C. 8^{-1}

   D. $16^{\frac{1}{2}}$

   E. $16^{-\frac{1}{2}}$

   F. 16^{0}

   1. $\frac{1}{8}$
   2. $\frac{1}{4}$
   3. $\frac{1}{2}$
   4. 1
   5. 2
   6. 4

4. Complete the table. Use powers of 27 in the top row and radicals or rational numbers in the bottom row.

27^1		$27^{\frac{1}{3}}$		$27^{-\frac{1}{2}}$	
27	$\sqrt{27}$		1		$\frac{1}{3}$

(From Unit 3, Lesson 3.)

5. What are the solutions to the equation $(x - 1)(x + 2) = -2$?

(From Unit 2, Lesson 11.)

6. Use exponent rules to explain why $(\sqrt{5})^3 = \sqrt{5^3}$.

(From Unit 3, Lesson 4.)

Lesson 6: Squares and Square Roots

- Let’s compare equations with squares and square roots.

6.1: Math Talk: Four Squares

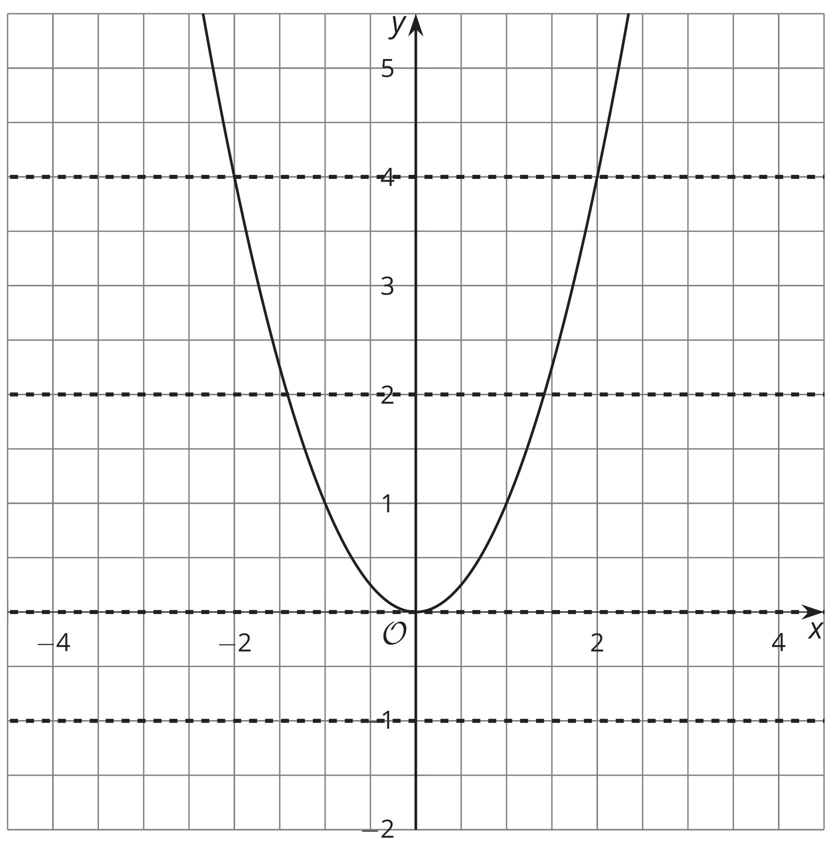
Find the solutions of each equation mentally.

$x^2 = 4$

$x^2 = 2$

$x^2 = 0$

$x^2 = -1$



6.2: Finding Square Roots

Clare was adding $\sqrt{4}$ and $\sqrt{9}$, and at first she wrote $\sqrt{4} + \sqrt{9} = 2 + 3$. But then she remembered that 2 and -2 both square to make 4, and that 3 and -3 both square to make 9. She wrote down all the possible combinations:

2 + 3 = 5
2 + (-3) = -1
(-2) + 3 = 1
(-2) + (-3) = -5

Then she wondered, “Which of these are the same as $\sqrt{4} + \sqrt{9}$? All of them? Or only some? Or just one?”

How would you answer Clare’s question? Give reasons that support your answer.

Are you ready for more?

1. How many solutions are there to each equation?
   a. $x^3 = 8$

   b. $y^3 = -1$

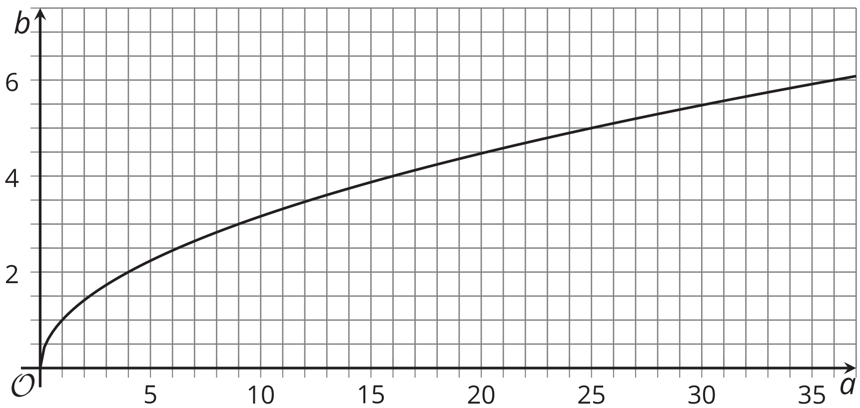
   c. $z^4 = 16$

   d. $w^4 = -81$

2. Write a rule to determine how many solutions there are to the equation $x^n = m$ where n and m are non-zero integers.

6.3: One Solution or Two?

1. The graph of $b = \sqrt{a}$ is shown.



a. Complete the table with the exact values and label the corresponding points on the graph with the exact values.

a	1	4	9	12	16	20
$\sqrt{a}$						

b. Label the point on the graph that shows the solution to $\sqrt{a} = 4$.

c. Label the point on the graph that shows the solution to $\sqrt{a} = 5$.

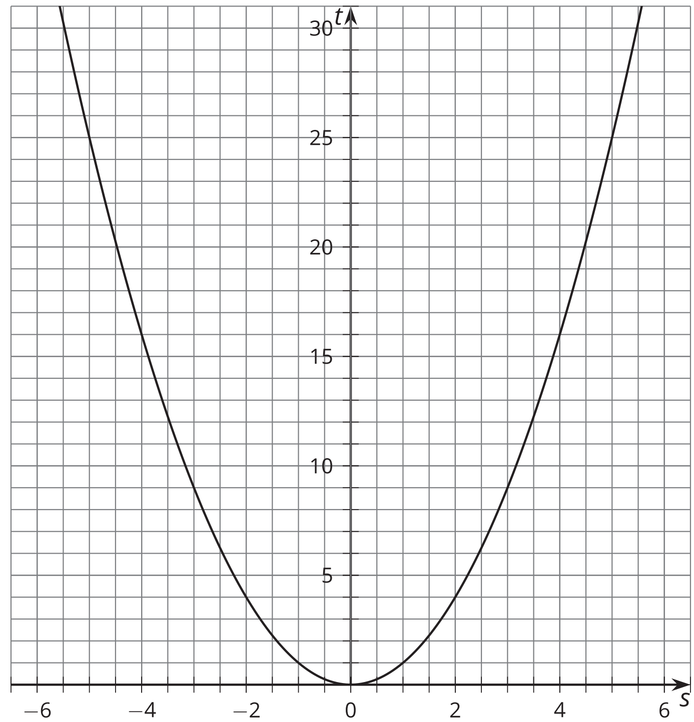
d. Label the point on the graph that shows the solution to $\sqrt{a} = \sqrt{5}$.

2. The graph of $t = s^2$ is shown.

a. Label the point(s) on the graph that show(s) the solution(s) to $s^2 = 25$.

b. Label the point(s) on the graph that show(s) the solution(s) to $\sqrt{t} = 5$.

c. Label the point(s) on the graph that show(s) the solution(s) to $s^2 = 5$.



Lesson 6 Summary

The symbol $\sqrt{11}$ represents the *positive* square root of 11. If we want to represent the negative square root, we write $-\sqrt{11}$.

The equation $x^2 = 11$ has two solutions, because $\sqrt{11}^2 = 11$, and also $\left(-\sqrt{11}\right)^2 = 11$.

The equation $\sqrt{x} = 11$ only has one solution, namely 121.

The equation $\sqrt{x} = \sqrt{11}$ only has one solution, namely 11.

The equation $\sqrt{x} = -11$ doesn’t have any solutions, because the left side is positive and the right side is negative, which is impossible, because a positive number cannot equal a negative number.

Lesson 6 Practice Problems

1. Select **all** solutions to the equation $x^2 = 7$.

   A. $\sqrt{7}$

   B. $-\sqrt{7}$

   C. 49

   D. -49

2. Find the solution(s) to each equation, if there are any.

   a. $x^2 = 9$

   b. $\sqrt{x} = 3$

   c. $\sqrt{x} = -3$

3. a. If c is a positive number, how many solutions does $x^2 = c$ have? Explain.

   b. If c is a positive number, how many solutions does $\sqrt{x} = c$ have? Explain.

4. Suppose that a friend missed class and never learned what $37^{\frac{1}{3}}$ means.

a. Use exponent rules your friend would already know to calculate $(37^{\frac{1}{3}})^3$.

b. Explain why this means that $37^{\frac{1}{3}}$ is the cube root of 37.

(From Unit 3, Lesson 3.)

5. Evaluate $8^{\frac{5}{3}}$.

6. Write each expression without using exponents.

a. $5^{\frac{2}{3}}$

b. $4^{-\frac{3}{2}}$

(From Unit 3, Lesson 5.)