Lesson 9: Solving Radical Equations

- Let’s practice solving radical equations.

9.1: Math Talk: Radical Equations

Solve these equations mentally:

$\sqrt[3]{x} = 1$

$\sqrt{7} = \sqrt{x - 1}$

$\sqrt{100} = 2x$

$\sqrt{x + 1} = -5$

9.2: Getting to the Root of the Problem

Find the solution(s) to each of these equations, or explain why there is no solution.

1. $\sqrt{a-5} = 5$

2. $\sqrt[3]{a-5} = 5$

3. $\sqrt[3]{b} = -2$

4. $\sqrt{c} + 2 = 0$

5. $\sqrt[3]{3-d} + 4 = 0$

6. $\sqrt{7} = \sqrt{x-1}$

7. $\sqrt{36} = 3y$

8. $22z = \sqrt[3]{11}$

9.3: Write Your Own Equation

1. Write an equation that includes a radical symbol with:
    a. one solution

    b. no solutions

    c. two solutions

2. Switch with a partner and solve their equations.

Are you ready for more?

Find all solutions to the equation $\sqrt{x} = \sqrt[3]{x}$. Explain how you know those are all of the solutions.

Lesson 9 Summary

Whenever we have an equation with a radical symbol that contains a variable, we can solve it by isolating the radical and then raising each side of the equation to a power in order to get a new equation without radicals. Here is an example:

$$\begin{aligned} -4 &= \sqrt[3]{5p+1} \\ (-4)^3 &= \left(\sqrt[3]{5p+1}\right)^3 \\ -64 &= 5p+1 \\ -65 &= 5p \\ -13 &= p \end{aligned}$$

Sometimes this results in an equation with solutions that do not make the original equation true. If we use this strategy, it is good to check the solutions to the new equation we got after raising each side to a power, to be sure they make the original equation true. In this example, we did find a solution to the original equation because $\sqrt[3]{5(-13)+1} = -4$.

Another way to solve these equations is to reason about what the answer is, instead of raising each side to a power. For example, if we are solving $\sqrt{1-x} + 5 = 11$, we can rearrange it to get $\sqrt{1-x} = 6$ and then think, "If the positive square root of $1-x$ is 6, then $1-x$ must be 36, since the positive square root of 36 is 6. So x must be -35, since $1-(-35) = 36$." If we check this result, we see that -35 is a solution to the original equation because $\sqrt{1-(-35)} + 5 = 11$.

Lesson 9 Practice Problems

1. Find the solution(s) to each of these equations, or explain why there is no solution.

   a. $\sqrt{x+5}+7=10$

   b. $\sqrt{x-2}+3=\text{-}2$

2. For each equation, decide how many solutions it has and explain how you know.

   a. $(x-4)^2=25$

   b. $\sqrt{x-4}=5$

   c. $x^3-7=\text{-}20$

   d. $6\cdot\sqrt[3]{x}=0$

3. Jada was solving the equation $\sqrt{6 - x} = -16$. She was about to square each side, but then she realized she could give an answer without doing any algebra. What did she realize?

4. Here are the steps Tyler took to solve the equation $\sqrt{x + 3} = -5$.

$$\begin{aligned} \sqrt{x+3} &= -5 \\ x + 3 &= 25 \\ x &= 22 \end{aligned}$$

a. Check Tyler's answer: Is the equation true if $x = 22$? Explain or show your reasoning.

b. What mistake did Tyler make?

5. Complete the table. Use powers of 16 in the top row and radicals or rational numbers in the bottom row.

16^1		$16^{\frac{1}{3}}$			16^{-1}
	4		1	$\frac{1}{4}$	$\frac{1}{16}$

(From Unit 3, Lesson 3.)

6. Which are the solutions to the equation $x^3 = 35$?

   A. $\sqrt[3]{35}$

   B. $-\sqrt[3]{35}$

   C. both $\sqrt[3]{35}$ and $-\sqrt[3]{35}$

   D. The equation has no solutions.

(From Unit 3, Lesson 8.)

Lesson 10: A New Kind of Number

- Let's invent a new number.

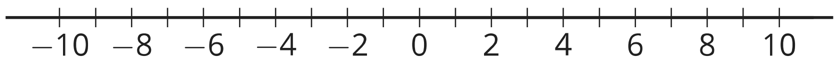
10.1: Numbers Are Inventions

Jada was helping her cousin with his math homework. He was supposed to solve the equation $8 + x = 5$. He said, "If I subtract 8 from both sides, I get $x = 5 - 8$. This doesn't make sense. You can't subtract a bigger number from a smaller number. If I have 5 grapes, I can't eat 8 of them!"

What do you think Jada could say to her cousin to help him understand why $5 - 8$ actually does make sense?

10.2: The Square Root of Negative One

Numbers on the number line are often called **real numbers**.



1. The equation $x^2 = 9$ has 2 real solutions. How can you see this on the graph of $y = x^2$? Draw points on this real number line to represent these 2 solutions.

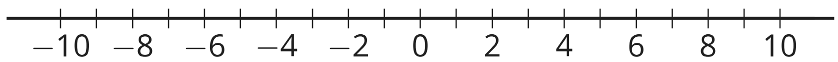
2. How many real solutions does $x^2 = 0$ have? Explain how you can see this on the graph of $y = x^2$. Draw the solution(s) on a real number line.

3. How many real solutions does $x^2 = -1$ have? Explain how you can see this on the graph of $y = x^2$. Draw the solution(s) on a real number line.

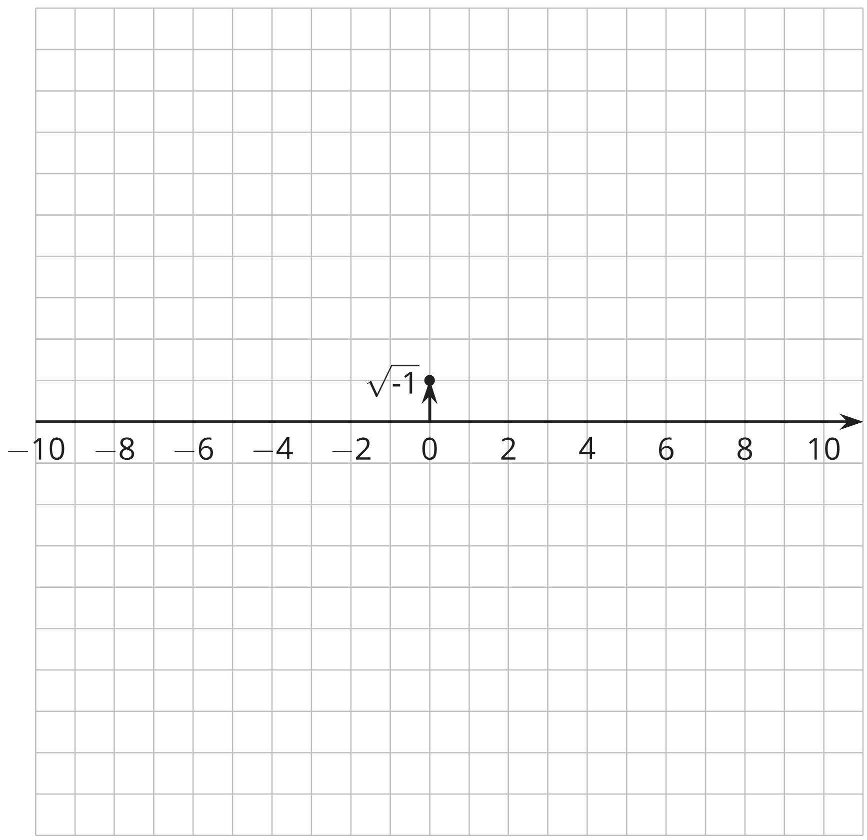
10.3: Imaginary Numbers

1. On the real number line:
    a. Draw an arrow starting at 0 that represents 3.

    b. Draw an arrow starting at 0 that represents -5.



2. This diagram shows an arrow that represents $\sqrt{-1}$.



    a. Draw an arrow starting at 0 that represents $3\sqrt{-1}$.

    b. Draw an arrow starting at 0 that represents $-\sqrt{-1}$.

    c. Draw an arrow starting at 0 that represents $-5\sqrt{-1}$.

Are you ready for more?

The absolute value of a real number is the length of the arrow that represents it.

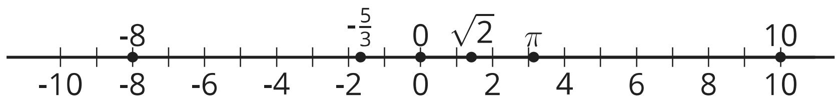
1. What is the relationship between the absolute value of a real number and the absolute value of the square of that number?

2. If we want $\sqrt{-1}$ and its square to have this same relationship, then what should the absolute value of $\sqrt{-1}$ be?

3. What should the absolute value of $3\sqrt{-1}$ be?

Lesson 10 Summary

Sometimes people call the number line the *real number line*.

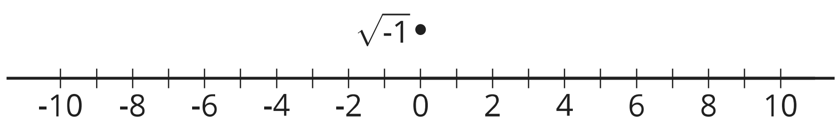


All **real numbers** are either positive, negative, or 0, and they can be plotted on this line. All the numbers we have used until this lesson have been real numbers. For real numbers, we know that:

- A positive number times a positive number is always positive. So when we square a positive number, the result will always be positive.
- A negative number times a negative number is always positive. So when we square a negative number, the result will always be positive.
- 0 squared is 0.

So squaring a *real* number never results in a negative number. We can conclude that the equation $x^2 = -1$ does not have any real number solutions. In other words, none of the numbers on the real number line satisfy this equation.

Mathematicians invented a new number that is *not* on the real number line. This new number was invented as a solution to the equation $x^2 = -1$. For now, let's write it $\sqrt{-1}$ and draw a point to represent this number. We can put it anywhere we want as long as we don't put it on the real number line. For example, let's put it here, right above 0 on the real number line:



This new number $\sqrt{-1}$ is a solution to the equation $x^2 = -1$, so $\left(\sqrt{-1}\right)^2 = -1$. If we draw a line that passes through 0 on the real number line and $\sqrt{-1}$, we get the *imaginary number line*. The numbers on the imaginary number line are called the **imaginary numbers**.

Glossary

- imaginary number
- real number

Lesson 10 Practice Problems

1. Select **all** the true statements.

   A. $\sqrt{-1}$ is an imaginary number.

   B. There are no real numbers that satisfy the equation $x = \sqrt{-1}$.

   C. Because $\sqrt{-1}$ is imaginary, no one does math with it.

   D. The equation $x^2 = -1$ has real solutions.

   E. $\sqrt{-1} = -1$ because $-1 \cdot -1 = -1$.

2. Plot each number on the real number line, or explain why the number is not on the real number line.

   a. $\sqrt{4}$

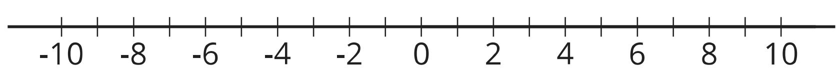
   b. $-\sqrt{4}$

   c. $\sqrt{-4}$

   d. $\sqrt{8}$

   e. $-\sqrt{8}$

   f. $\sqrt{-8}$



3. Explain why $(x-4)^2 = -9$ has no real solutions.

4. Which value is closest to $10^{-\frac{1}{2}}$?

A. -5

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. 3

(From Unit 3, Lesson 5.)

5. Which is a solution to the equation $\sqrt{6 - x} + 5 = 10$?

A. -19

B. 19

C. 21

D. The equation has no solutions.

(From Unit 3, Lesson 7.)

6. Select **all** equations for which -64 is a solution.

A. $\sqrt{x} = 8$

B. $\sqrt{x} = -8$

C. $\sqrt[3]{x} = 4$

D. $\sqrt[3]{x} = -4$

E. $-\sqrt{x} = 8$

F. $\sqrt{-x} = 8$

(From Unit 3, Lesson 8.)