Lesson 8: Unknown Exponents

- Let's find unknown exponents.

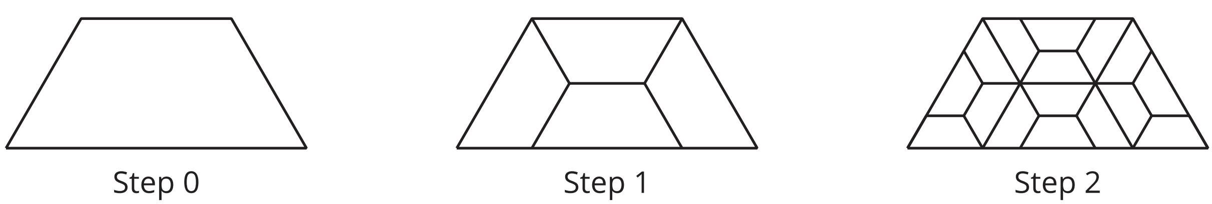
8.1: A Bunch of x's

Solve each equation. Be prepared to explain your reasoning.

1. $\frac{x}{3} = 12$
2. $3x^2 = 12$
3. $x^3 = 12$
4. $\sqrt[3]{x} = 12$
5. $\sqrt{3x} = 12$
6. $\frac{3}{x} = 12$

8.2: A Tessellated Trapezoid

Here is a pattern showing a trapezoid being successively decomposed into four similar trapezoids at each step.



1. If n is the step number, how many of the smallest trapezoids are there when n is 4? What about when n is 10?

2. At a certain step, there are 262,144 smallest trapezoids.
    a. Write an equation to represent the relationship between n and the number of trapezoids in that step.

    b. Explain to a partner how you might find the value of that step number.

8.3: Successive Splitting

In a lab, a colony of 100 bacteria is placed on a petri dish. The population triples every hour.

1. How would you estimate or find the population of bacteria in:
    a. 4 hours?

    b. 90 minutes?

    c. $\frac{1}{2}$ hour?

2. How would you estimate or find the number of hours it would take the population to grow to:
    a. 1,000 bacteria?

    b. double the initial population?

Are you ready for more?

A $1,000 investment increases in value by 5% each year. About how many years does it take for the value of the investment to double? Explain how you know.

8.4: Missing Values

Complete the tables.

x			-1	0	$\frac{1}{2}$	1			5		
2^x	$\frac{1}{32}$	$\frac{1}{4}$	$\frac{1}{2}$				4	16		256	1,024

x				$\frac{1}{3}$	$\frac{1}{2}$				
5^x	$\frac{1}{25}$	$\frac{1}{5}$	1			5	125	625	3,125

Be prepared to explain how you found the missing values.

Lesson 8 Summary

Sometimes we know the value of an exponential expression but we don't know the exponent that produces that value.

For example, suppose the population of a town was 1 thousand. Since then, the population has doubled every decade and is currently at 32 thousand. How many decades has it been since the population was 1 thousand?

If we say that d is the number of decades since the population was 1 thousand, then $1 \cdot 2^d$, or just 2^d, represents the population, in thousands, after d decades. To answer the question, we need to find the exponent in $2^d = 32$. We can reason that since $2^5 = 32$, it has been 5 decades since the population was 1 thousand people.

When did the town have 250 people? Assuming that the doubling started before the population was measured to be 1 thousand, we can write: $2^d = 0.25$ or $2^d = \frac{1}{4}$. We know that $2^{-2} = \frac{1}{4}$, so the exponent d has a value of -2. The population was 250 two decades before it was 1,000.

But it may not always be so straightforward to calculate. For example, it is harder to tell the value of d in $2^d = 805$ or in $2^d = 4.5$. In upcoming lessons, we'll learn more ways to find unknown exponents.

Lesson 8 Practice Problems

1. A pattern of dots grows exponentially. The table shows the number of dots at each step of the pattern.

step number	0	1	2	3
number of dots	1	5	25	125

   a. Write an equation to represent the relationship between the step number, n, and the number of dots, y.

   b. At one step, there are 9,765,625 dots in the pattern. At what step number will that happen? Explain how you know.

2. A bacteria population is modeled by the equation $p(h) = 10{,}000 \cdot 2^h$, where h is the number of hours since the population was measured.

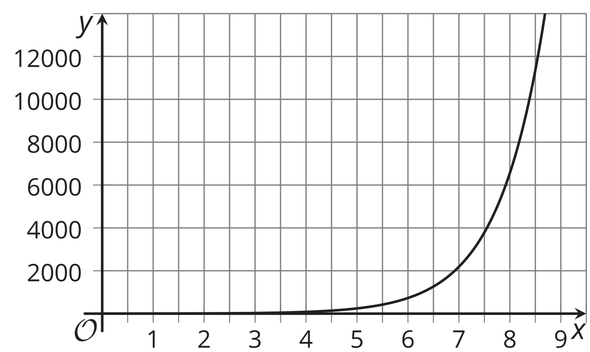
   About how long will it take for the population to reach 100,000? Explain your reasoning.

3. Complete the table.

x			-2	0	$\frac{1}{3}$	1		
10^x	$\frac{1}{10{,}000}$	$\frac{1}{1{,}000}$	$\frac{1}{100}$				1,000	1,000,000,000

4. Here is a graph of $y = 3^x$.

   What is the approximate value of x satisfying $3^x = 10{,}000$? Explain how you know.

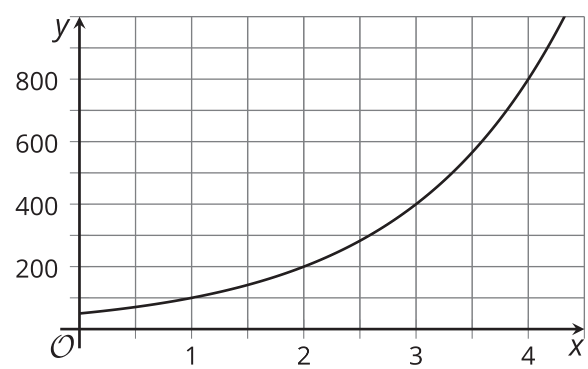


5. One account doubles every 2 years. A second account triples every 3 years. Assuming the accounts start with the same amount of money, which account is growing more rapidly?

6. How would you describe the output of this graph for:

   a. inputs from 0 to 1

   b. inputs from 3 to 4



(From Unit 4, Lesson 1.)

7. The half-life of carbon-14 is about 5730 years.

   a. Complete the table, which shows the amount of carbon-14 remaining in a plant fossil at the different times since the plant died.

   b. About how many years will it be until there is 0.1 picogram of carbon-14 remaining in the fossil? Explain how you know.

years	picograms
0	3
5730	
$2 \cdot 5730$	
$3 \cdot 5730$	
$4 \cdot 5730$	

(From Unit 4, Lesson 7.)

Lesson 9: What is a Logarithm?

- Let's learn about logarithms.

9.1: Math Talk: Finding Solutions

Find or estimate the value of each variable mentally.

$4^a = 16$

$4^b = 2$

$4^{\frac{5}{2}} = c$

$4^d = 56$

9.2: A Table of Numbers

x	$\log_{10}(x)$
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1

x	$\log_{10}(x)$
20	1.3010
30	1.4771
40	1.6021
50	1.6990
60	1.7782
70	1.8451
80	1.9031
90	1.9542
100	2

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3

x	$\log_{10}(x)$
2,000	3.3010
3,000	3.4771
4,000	3.6021
5,000	3.6990
6,000	3.7782
7,000	3.8451
8,000	3.9031
9,000	3.9542
10,000	4

1. Analyze the table and discuss with a partner what you think the table tells us.

2. Use the table to find the value of the unknown exponent that makes each equation true.
   a. $10^w = 1{,}000$
   b. $10^y = 9$
   c. $10^z = 90$

3. Notice that some values in the columns labeled $\log_{10} x$ are whole numbers, but most are decimals. Why do you think that is?

9.3: Hello, Logarithm!

1. Here are two true equations based on the information from the table:

$$\log_{10} 100 = 2$$
$$\log_{10} 1{,}000 = 3$$

   What values could replace the "?" in these equations to make them true?

   a. $\log_{10} 1{,}000{,}000 = ?$

   b. $\log_{10} 1 = ?$

   c. $\log_{10} ? = 4$

2. Between which two whole numbers is the value of $\log_{10} 600$? Be prepared to explain how do you know.

3. The term *log* is short for **logarithm**. Discuss the following questions with a partner and record your responses:
   a. What do you think logarithm means or does?

   b. Next to "log" is a subscript—a number or letter printed smaller and below the line of text. What do you think the subscript tells us?

   c. What about the other two numbers on either side of the equal sign (for example, the 100 and the 2 in $\log_{10} 100 = 2$)? What do they tell us?

Are you ready for more?

1. For which whole number values of n is $\log_{10}(n)$ an integer?

2. Why will $\log_{10}(n)$ never be equal to a non-integer rational number?

Lesson 9 Summary

We know how to solve equations such as $10^a = 10{,}000$ or $10^b = \frac{1}{100}$ by thinking about integer powers of 10. The solutions are $a = 4$ and $b = -2$. What about an equation such as $10^p = 250$?

Because $10^2 = 100$ and $10^3 = 1{,}000$, we know that p is between 2 and 3. We can use a **logarithm** to represent the exact solution to this equation and write it as:

$$p = \log_{10} 250$$

The expression is read "the log, base 10, of 250."

- The small, slightly lowered "10" refers to the base of 10.
- The 250 is the value of the power of 10.
- $\log_{10} 250$ is the value of the exponent p that makes 10^p equal 250.

Base 10 logarithms are often written without the number 10. So $\log_{10} 250$ can also be written as $\log 250$ and this expression is read "the log of 250."

One way to estimate logarithms is with a logarithm table. For example, using this base 10 logarithm table we can see that $\log_{10} 250$ is between 2.3 and 2.48.

x	$\log_{10}(x)$
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1

x	$\log_{10}(x)$
20	1.3010
30	1.4771
40	1.6021
50	1.6990
60	1.7782
70	1.8451
80	1.9031
90	1.9542
100	2

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3

x	$\log_{10}(x)$
2,000	3.3010
3,000	3.4771
4,000	3.6021
5,000	3.6990
6,000	3.7782
7,000	3.8451
8,000	3.9031
9,000	3.9542
10,000	4

Glossary

- logarithm

Lesson 9 Practice Problems

1. For each equation in the left column, find in the right column an exact or approximate value for the unknown exponent so that the equation is true.

A. $10^y = 10$	1. 0.602
B. $10^y = 20$	2. -1
C. $10^y = 2{,}000$	3. 1
D. $10^y = 900$	4. 2.954
E. $10^y = 4$	5. 1.301
	6. 3.301
	7. 1.999

2. Here is a logarithmic expression: $\log_{10} 100$.

   a. How do we say the expression in words?

   b. Explain in your own words what the expression means.

   c. What is the value of this expression?

3. The base 10 log table shows that the value of $\log_{10} 50$ is about 1.69897. Explain or show why it makes sense that the value is between 1 and 2.

4. Here is a table of some logarithm values.

   a. What is the approximate value of $\log_{10}(400)$?

   b. What is the value of $\log_{10}(1000)$? Is this value approximate or exact? Explain how you know.

x	$\log_{10}(x)$
200	2.3010
300	2.4771
400	2.6021
500	2.6990
600	2.7782
700	2.8451
800	2.9031
900	2.9542
1,000	3

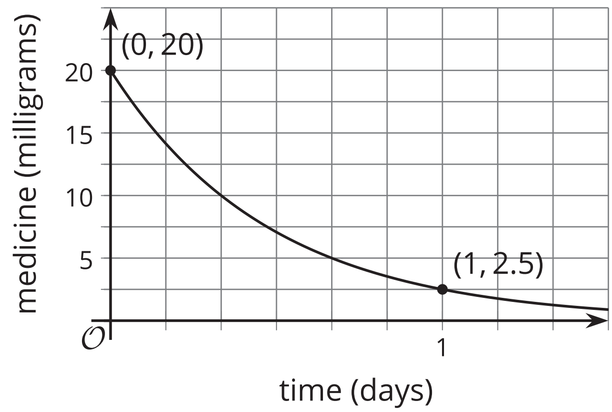
5. What is the value of $\log_{10}(1{,}000{,}000{,}000)$? Explain how you know.

6. A bank account balance, in dollars, is modeled by the equation $f(t) = 1{,}000 \cdot (1.08)^t$, where t is time measured in years.

   About how many years will it take for the account balance to double? Explain or show how you know.

   (From Unit 4, Lesson 8.)

7. The graph shows the number of milligrams of a chemical in the body, d days after it was first measured.



a. Explain what the point $(1, 2.5)$ means in this situation.

b. Mark the point that represents the amount of medicine left in the body after 8 hours.

(From Unit 4, Lesson 3.)

8. The exponential function f takes the value 10 when $x = 1$ and 30 when $x = 2$.

a. What is the y-intercept of f? Explain how you know.

b. What is an equation defining f?

(From Unit 4, Lesson 6.)