Lesson 7: Inequivalent Equations

- Let's see what happens when we square each side of an equation.

7.1: 2 and -2

What do you notice? What do you wonder?

- $x^2 = 4$
- $x^2 = -4$
- $(x-2)(x+2) = 0$
- $x = \sqrt{4}$

7.2: Careful When You Take the Square Root

Tyler was solving this equation:

$$x^2 - 1 = 3$$

He said, "I can add 1 to each side of the equation and it doesn't change the equation. I get $x^2 = 4$."

1. Priya said, "It does change the equation. It just doesn't change the solutions!" Then she showed these two graphs.

Figure A

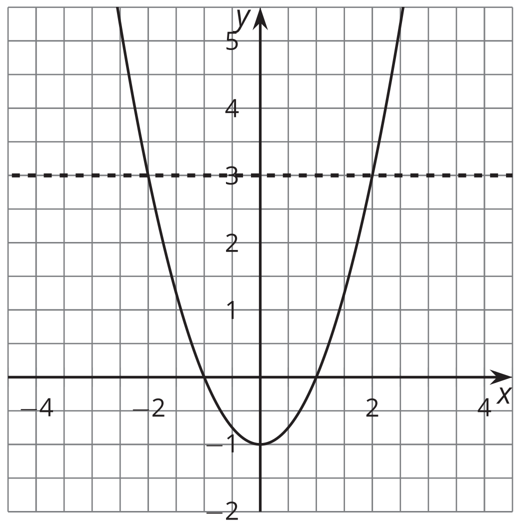
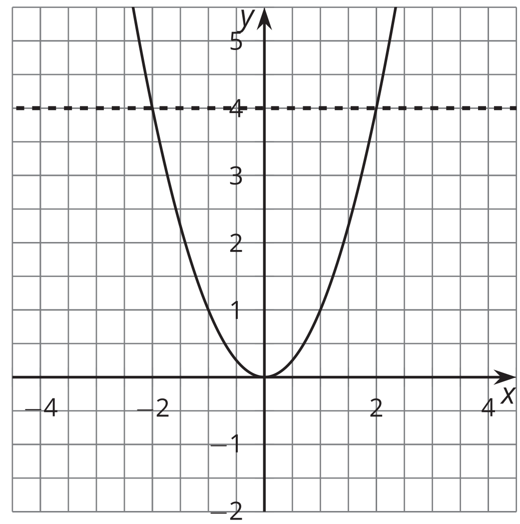


Figure B



a. How can you see the solution to the equation $x^2 - 1 = 3$ in Figure A?

b. How can you see the solution to the equation $x^2 = 4$ in Figure B?

c. Use the graphs to explain why the equations have the same solutions.

2. Tyler said, "Now I can take the square root of each side to get the solution to $x^2 = 4$. The square root of x^2 is x. The square root of 4 is 2." He wrote:

$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = 2$$

Priya said, "But the graphs show that there are *two* solutions!" What went wrong?

7.3: Another Way to Solve

Han was solving this equation:

$$\frac{x+3}{2}=4$$

He said, "I know that half of $x+3$ is 4. So $x+3$ must be 8, since half of 8 is 4. This means that x is 5."

1. Use Han's reasoning to solve this equation: $(x+3)^2 = 4$.

2. What advice would you give to someone who was going to solve an equation like $(x+3)^2 = 4$?

7.4: What Happens When You Square Each Side?

Mai was solving this equation:

$$\sqrt{x-1} = 3$$

She said, “I can square each side of the equation to get another equation with the same solutions.” Then she wrote:

$$\begin{aligned} \sqrt{x-1} &= 3 \\ (\sqrt{x-1})^2 &= 3^2 \\ x-1 &= 9 \\ x &= 10 \end{aligned}$$

1. Check to see if her solution makes the original equation true.

2. Andre said, “I tried your technique to solve

   $$\sqrt{x-1} = \text{-}3$$

   but it didn’t work.” Why doesn’t it work? Explain or show your reasoning.

7.5: Solve These Equations With Square Roots in Them

Find the solution(s) to each of these equations, or explain why there is no solution.

1. $\sqrt{t+4} = 3$

2. $-10 = -\sqrt{a}$

3. $\sqrt{3-w} - 4 = 0$

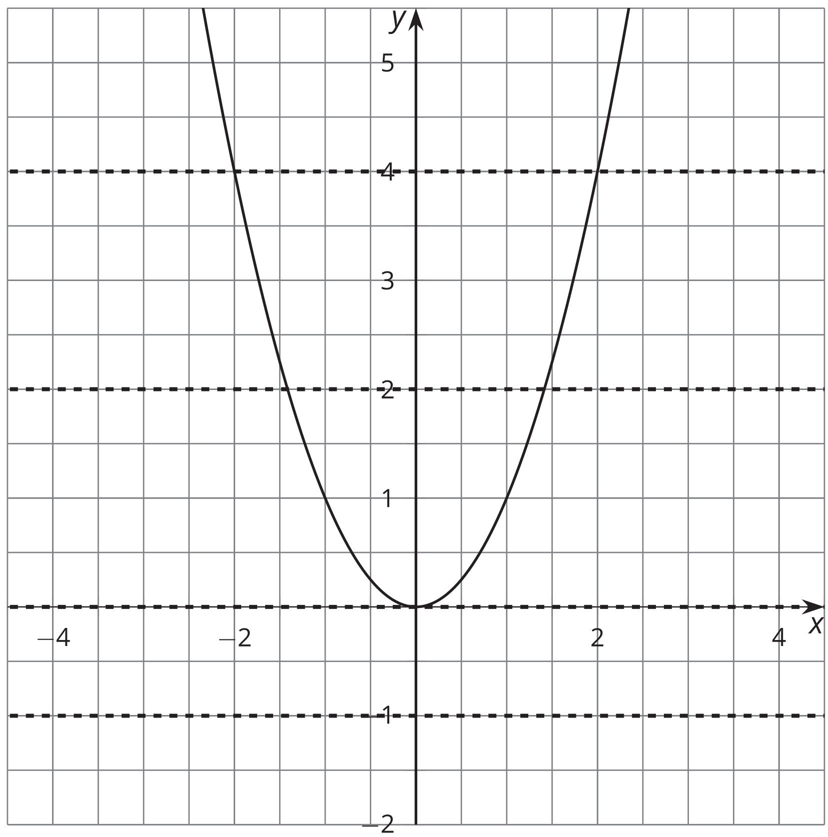
4. $\sqrt{z} + 9 = 0$

Are you ready for more?

Are there values of a and b so that the equation $\sqrt{t+a} = b$ has more than one solution? Explain your reasoning.

Lesson 7 Summary

Every positive number has *two* square roots. You can see this by looking at the graph of $y = x^2$:



If y is a positive number like 4, then we can see that $y = 4$ crosses the graph in two places, so the equation $x^2 = 4$ will have two solutions, namely, $\sqrt{4}$ and -$\sqrt{4}$. This is true for any positive number a: $y = a$ will cross the graph in two places, and $x^2 = a$ will have two solutions, $x = \sqrt{a}$ and $x = -\sqrt{a}$.

When we have a square root in an equation like $\sqrt{t} - 6 = 0$, we can isolate the square root and then square each side:

$$\begin{aligned} \sqrt{t} - 6 &= 0 \\ \sqrt{t} &= 6 \\ t &= 6^2 \\ t &= 36 \end{aligned}$$

But sometimes, squaring each side of an equation gives results that aren't solutions to the original equation. For example:

$$\begin{aligned} \sqrt{t} + 6 &= 0 \\ \sqrt{t} &= -6 \\ t &= (-6)^2 \\ t &= 36 \end{aligned}$$

Note that 36 is *not* a solution to the original equation, because $\sqrt{36} + 6$ doesn't equal 0. In fact, $\sqrt{t} + 6 = 0$ has no solutions, because it's impossible for the sum of two positive numbers to be zero.

Remember: sometimes the new equation has solutions that the old equation doesn't have. Always check your solutions in the original equation!

Lesson 7 Practice Problems

1. Noah solved the equation $5x^2 = 45$. Here are his steps:

$$5x^2 = 45$$
$$x^2 = 9$$
$$x = 3$$

Do you agree with Noah? Explain your reasoning.

2. Find the solution(s) to each equation, or explain why there is no solution.

    a. $\sqrt{x+4} + 7 = 5$

    b. $\sqrt{47 - x} - 2 = 4$

    c. $\frac{1}{2}\sqrt{20 + x} = 5$

3. Which is a solution to the equation $\sqrt{5 - x} + 13 = 4$?

A. 86

B. 81

C. 9

D. The equation has no solution.

4. Select **all** expressions that are equal to $\frac{1}{(\sqrt{2})^5}$.

A. $-\frac{5}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2^5}}$

C. $\frac{1}{\sqrt{32}}$

D. $-(\sqrt{2})^5$

E. $-2^{\frac{5}{2}}$

F. $2^{-\frac{5}{2}}$

(From Unit 3, Lesson 5.)

5. Which are the solutions to the equation $x^2 = 36$?

A. 6 only

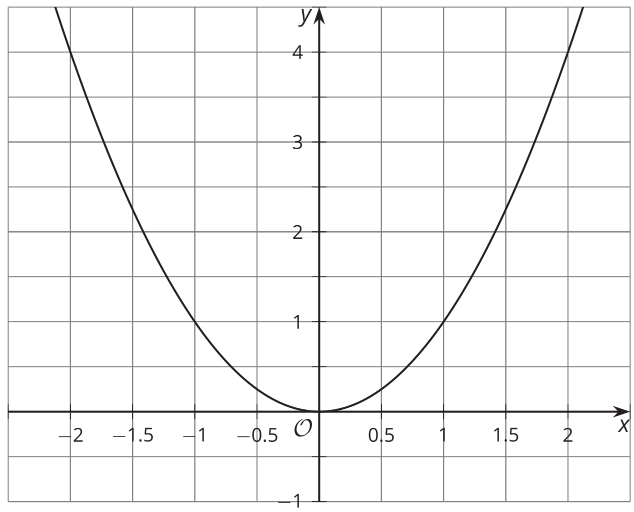
B. -6 only

C. 6 and -6

D. This equation has no solutions.

(From Unit 3, Lesson 6.)

6. Here is a graph of $y = x^2$.



a. Use the graph to estimate all solutions to the equation $x^2 = 3$.

b. If you square your estimates, what number should they be close to?

c. Square your estimates. How close did you get to this number?

(From Unit 3, Lesson 6.)

7. The polynomial function $q(x) = 3x^3 + 11x^2 - 14x - 40$ has a known factor of $(3x + 5)$. Rewrite $q(x)$ as the product of linear factors.

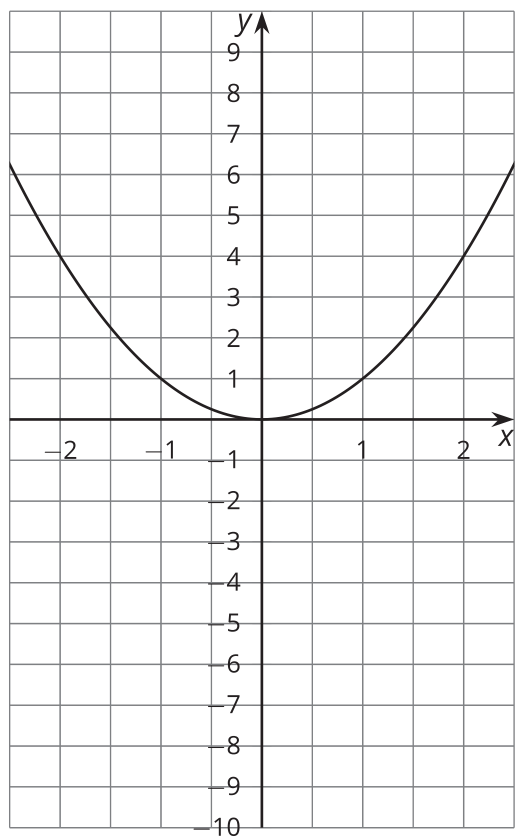
(From Unit 2, Lesson 12.)

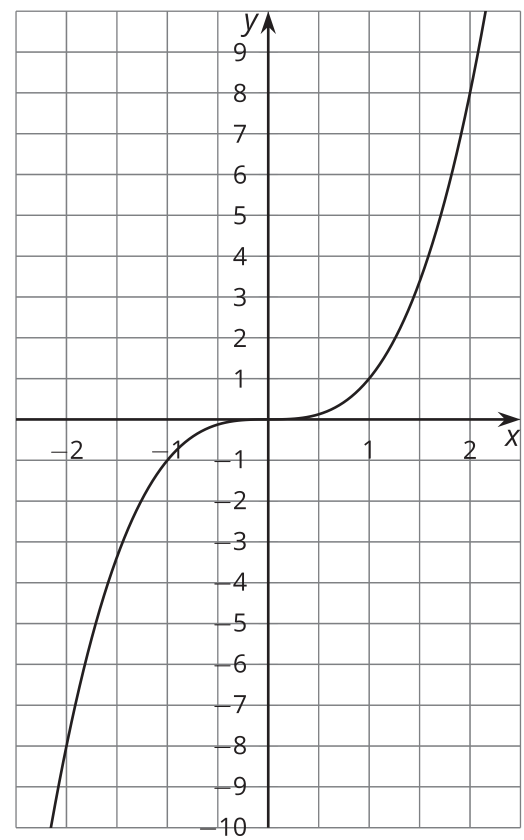
Lesson 8: Cubes and Cube Roots

- Let’s compare equations with cubes and cube roots.

8.1: Put Your Arm(s) Up

How are these graphs the same? How are they different?





8.2: Finding Cube Roots with a Graph

How many solutions are there to each of the following equations? Estimate the solution(s) from the graph of $y = x^3$. Check your estimate by substituting it back into the equation.

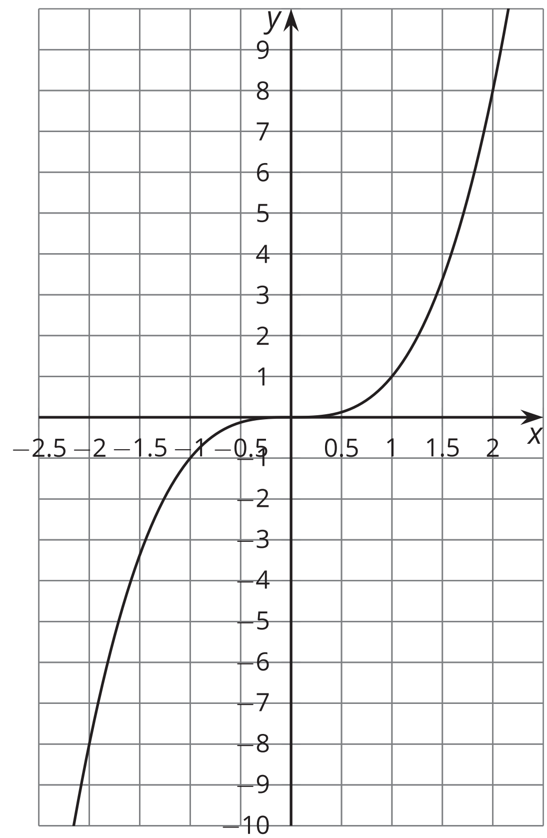
1. $x^3 = 8$

2. $x^3 = 2$

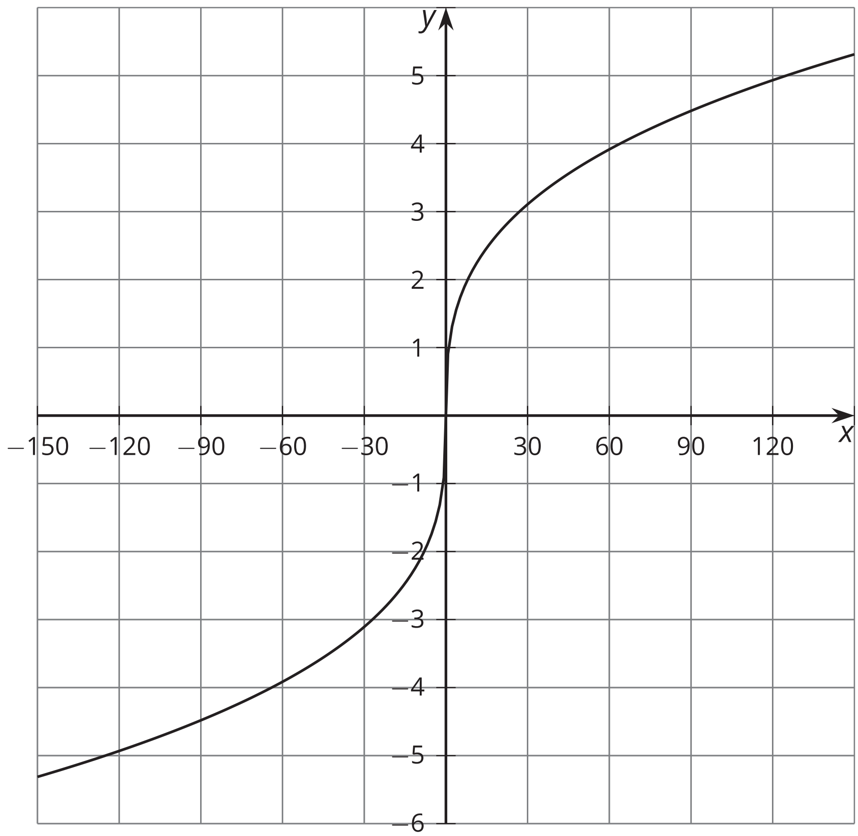
3. $x^3 = 0$

4. $x^3 = -8$

5. $x^3 = -2$



8.3: Cube Root Equations



1. Use the graph of $y = \sqrt[3]{x}$ to estimate the solution(s) to $\sqrt[3]{x} = -4$.

2. Use the meaning of cube roots to find an exact solution to the equation $\sqrt[3]{x} = -4$. How close was your estimate?

3. Find the solution of the equation $\sqrt[3]{x} = 3.5$ using the meaning of cube roots. Use the graph to check that your solution is reasonable.

8.4: Solve These Equations With Cube Roots in Them

Here are a lot of equations:

- $\sqrt[3]{t+4} = 3$
- $-10 = -\sqrt[3]{a}$
- $\sqrt[3]{3-w} - 4 = 0$
- $\sqrt[3]{z} + 9 = 0$
- $\sqrt[3]{r^3 - 19} = 2$
- $5 - \sqrt[3]{k+1} = -1$
- $\sqrt[3]{p+4} - 2 = 1$
- $6 - \sqrt[3]{b} = 0$
- $\sqrt[3]{2n} + 3 = -5$
- $4 + \sqrt[3]{-m} + 4 = 6$
- $-\sqrt[3]{c} = 5$
- $\sqrt[3]{s-7} + 3 = 0$

1. Without solving, identify 3 equations that you think would be the least difficult to solve and 3 equations that you think would be the most difficult to solve. Be prepared to explain your reasoning.

2. Choose 4 equations and solve them. At least one should be from your “least difficult” list and at least one should be from your “most difficult” list.

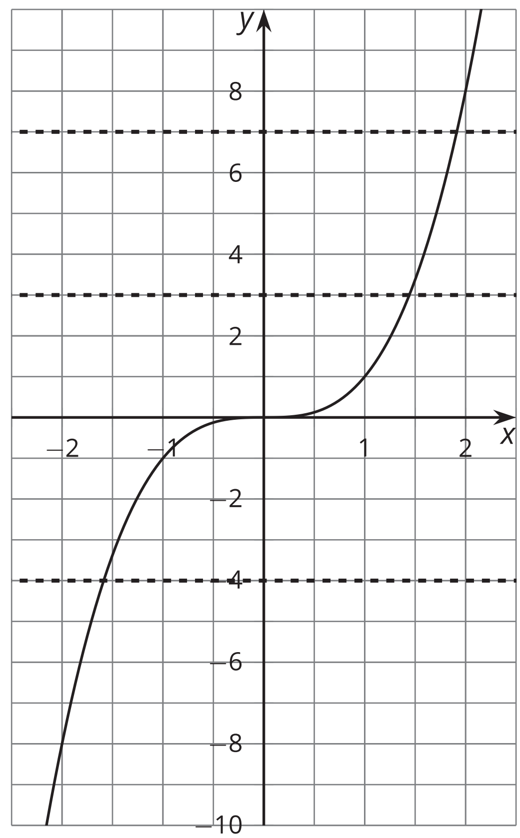
Are you ready for more?

All of these equations were equivalent to equations that could be written in the form $\sqrt[3]{ax+b}+c=0$ for some constants a, b, and c. Find a formula that would solve any such equation for x in terms of a, b, and c.

Lesson 8 Summary

Every number has exactly one cube root. You can see this by looking at the graph of $y = x^3$.

If y is any number, for example, -4, then we can see that $y =$ -4 crosses the graph in one and only one place, so the equation $x^3 =$ -4 will have the solution $-\sqrt[3]{4}$. This is true for any number a: $y = a$ will cross the graph in exactly one place, and $x^3 = a$ will have one solution, $\sqrt[3]{a}$.



In an equation like $\sqrt[3]{t} + 6 = 0$, we can isolate the cube root and cube each side:

$$\sqrt[3]{t} + 6 = 0$$
$$\sqrt[3]{t} = -6$$
$$t = (-6)^3$$
$$t = -216$$

While cubing each side of an equation won't create an equation with solutions that are different than the original equation, it is still a good idea to always check solutions in the original equation because little mistakes can creep in along the way.

Lesson 8 Practice Problems

1. Select **all** equations for which -3 is a solution.

   A. $x^2 = 9$

   B. $x^2 = -9$

   C. $x^3 = 27$

   D. $x^3 = -27$

   E. $-x^2 = 9$

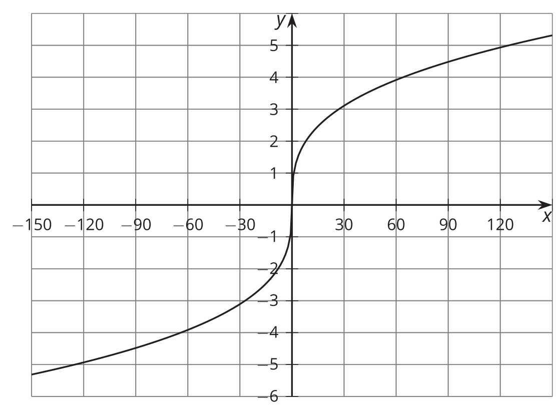
   F. $(-x)^2 = 9$

2. a. Use the graph of $y = \sqrt[3]{x}$ to estimate the solution(s) to the following equations.

   i. $\sqrt[3]{x} = 2$

   ii. $\sqrt[3]{x} = -4.5$

   iii. $\sqrt[3]{x} = 3.75$



   b. Use the meaning of cube roots to find exact solutions to all three equations.

3. Which are the solutions to the equation $x^3 = -125$?

   A. 5

   B. -5

   C. both 5 and -5

   D. The equation has no solutions.

4. Complete the table. Use powers of 16 in the top row. Use radicals or rational numbers in the second row.

	$16^{-\frac{3}{4}}$		$16^{-\frac{1}{4}}$	
$\frac{1}{16}$		$\frac{1}{4}$		1

(From Unit 3, Lesson 5.)

5. Which are the solutions to the equation $\sqrt{x} = -8$?

A. 64 only

B. -64 only

C. 64 and -64

D. This equation has no solutions.

(From Unit 3, Lesson 6.)

6. Find the solution(s) to each equation, or explain why there is no solution.

a. $x^2 + 6 = 55$

b. $x^2 + 16 = 0$

c. $x^2 - 3.25 = 21.75$

(From Unit 3, Lesson 7.)