Lesson 6: Different Forms

- Let's use the different forms of polynomials to learn about them.

6.1: Which One Doesn't Belong: Small Differences

Which one doesn't belong?

A: $y = (x + 4)(x - 6)$

B: $y = 2x^2 - 8x - 24$

C: $y = x^2 + 5x - 25$

D: $y = x^3 + 3x^2 - 10x - 24$

6.2: The Return of the Box

Earlier, we learned we can make a box from a piece of paper by cutting squares of side length x from each corner and then folding up the sides. Let's say we now have a piece of paper that is 8.5 inches by 14 inches. The volume V, in cubic inches, of the box is a function of the side length x where $V(x) = (14 - 2x)(8.5 - 2x)(x)$.

1. Identify the degree and leading term of the polynomial. Explain or show your reasoning.

2. Without graphing, what can you say about the horizontal and vertical intercepts of the graph of V? Do these points make sense in this situation?

6.3: Using Diagrams to Multiply

1. Use the distributive property to show that each pair of expressions is equivalent.
   a. $(x+2)(x+4)$ and x^2+6x+8

   b. $(x+6)(x+\text{-}5)$ and x^2+x-30

   c. $(x^2+10x+7)(2x-1)$ and $2x^3+19x^2+4x-7$

   d. $(4x^3-8)(x^2+3)$ and $4x^5+12x^3-8x^2-24$

2. Write a pair of expressions that each have 2 or 3 terms, and trade them with your partner. Multiply the expressions they gave you.

6.4: Spot the Differences

Let $f(x) = (x - 2)(x + 3)(x - 7)$ and $g(x) = \frac{1}{2}(x - 2)(x + 3)(x - 7)$.

1. Use graphing technology to graph both functions in the same window of $-10 \leq x \leq 10$ and $-100 \leq y \leq 100$. Describe how the two graphs are the same and how they are different.

2. What degree do these polynomials have? Rewrite each expression in standard form to check.

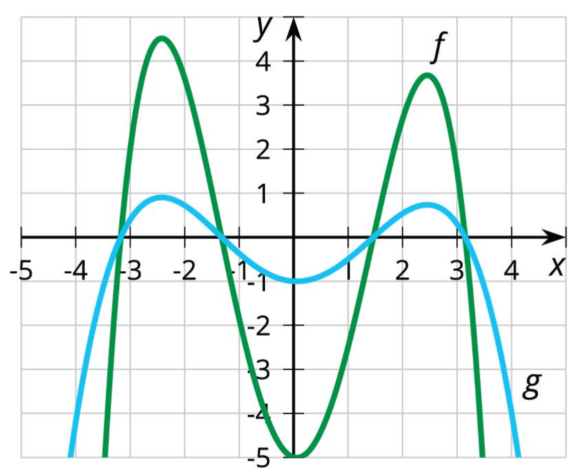
3. Let $h(x) = (3x - 6)(x + 3)(x - 7)$. What do you think the graph of $y = h(x)$ will look like compared to $y = f(x)$? Use graphing technology to check your prediction.

Are you ready for more?

Here are the graphs of two polynomial functions, f and g. We know that $g(x) = k \cdot f(x)$.

1. Why do the two graphs have different vertical intercepts but the same horizontal intercepts?

2. What is the value of k?



Lesson 6 Summary

We can express polynomials in different, equivalent, algebraic forms. These forms can give us different information about features of the polynomial and its graph. Earlier, we learned about expressing a polynomial function in factored form to identify zeros. The standard form of a polynomial, that is, the expanded version of factored form, makes it easier to identify different information about a polynomial.

For example, here are the expressions for a polynomial $P(x)$ written in factored form and standard form:

$$P(x) = 0.25(x-1)^2(x+2)(x-3)(x+3)$$

$$P(x) = 0.25x^5 - 3x^3 + 0.5x^2 + 6.75x - 4.5$$

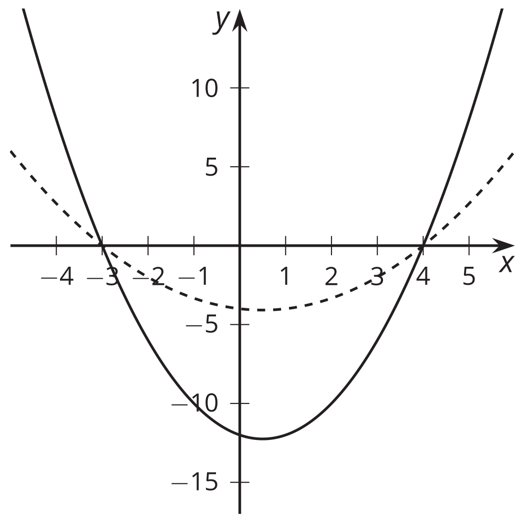
In standard form, two key features of the polynomial function can be identified: the constant term and the degree.

The constant term, shown as -4.5 in the example, tells us the value of the function when $x = 0$. In a graph of the function, this point is known as the vertical intercept.

The degree, shown as 5 in the example, tells us about the general shape of the graph of the polynomial, which is something we'll learn more about in future lessons.

Lesson 6 Practice Problems

1. $f(x) = (x + 3)(x - 4)$ and $g(x) = \frac{1}{3}(x + 3)(x - 4)$. The graphs of each are shown here.



   a. Which graph represents which polynomial function? Explain how you know.

2. For each polynomial function, rewrite the polynomial in standard form. Then state its degree and constant term.

   a. $f(x) = (x + 1)(x + 3)(x - 4)$

   b. $g(x) = 3(x + 1)(x + 3)(x - 4)$

3. Tyler incorrectly says that the constant term of $(x+4)(x-4)$ is zero.

   a. What is the correct constant term?

   b. What is Tyler's mistake? Explain your reasoning.

4. Which of these standard form equations is equivalent to $(x+1)(x-2)(x+4)(3x+7)$?

   A. $x^4+10x^3+15x^2-50x-56$

   B. $x^4+10x^3+15x^2-50x+56$

   C. $3x^4+16x^3+3x^2-66x-56$

   D. $3x^4+16x^3+3x^2-66x+56$

5. Select **all** polynomial expressions that are equivalent to $5x^3+7x-4x^2+5$.

   A. $13x^5$

   B. $5x^3-4x^2+7x+5$

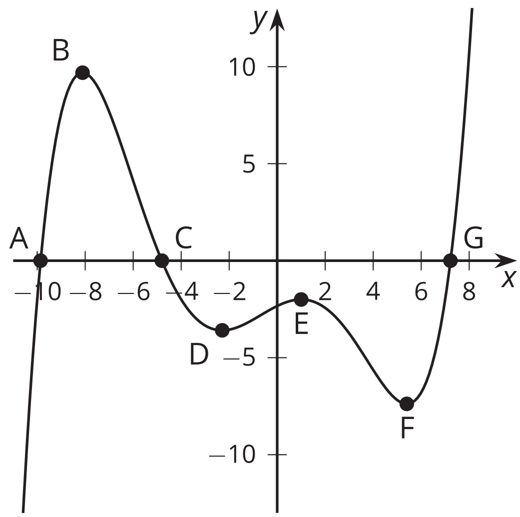
   C. $5x^3+4x\cdot 2+7x+5$

   D. $5+4x-7x^2+5x^3$

   E. $5+7x-4x^2+5x^3$

   (From Unit 2, Lesson 2.)

6. Select **all** the points which are relative minimums of this graph of a polynomial function.



A. Point A

B. Point B

C. Point C

D. Point D

E. Point E

F. Point F

G. Point G

(From Unit 2, Lesson 3.)

7. What are the x-intercepts of the graph of $y = (3x + 8)(5x - 3)(x - 1)$?

(From Unit 2, Lesson 5.)

Lesson 7: Using Factors and Zeros

- Let's write some polynomials.

7.1: More Than Factors

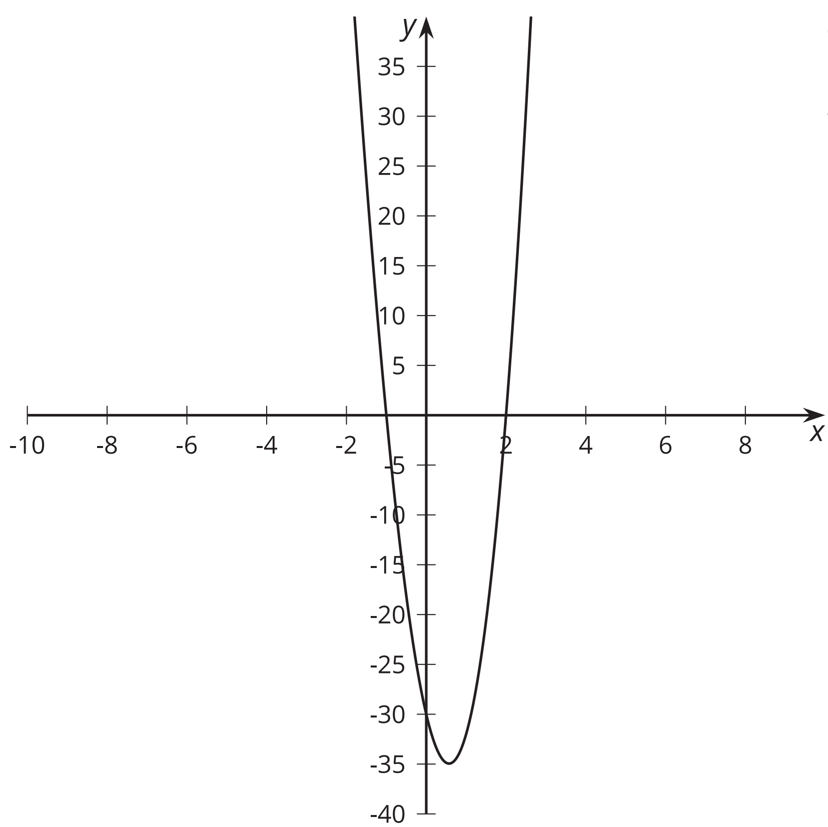
M and K are both polynomial functions of x where
$M(x) = (x+3)(2x-5)$ and $K(x) = 3(x+3)(2x-5)$.

1. How are the two functions alike? How are they different?

2. If a graphing window of -$5 \leq x \leq 5$ and -$20 \leq y \leq 20$ shows all intercepts of a graph of $y = M(x)$, what graphing window would show all intercepts of $y = K(x)$?

7.2: Choosing Windows

Mai graphs the function p given by $p(x) = (x+1)(x-2)(x+15)$ and sees this graph.



She says, "This graph looks like a parabola, so it must be a quadratic."

1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

Are you ready for more?

Select some different windows for graphing the function $q(x) = 23(x-53)(x-18)(x+111)$. What is challenging about graphing this function?

7.3: What's the Equation?

Write a possible equation for a polynomial whose graph has the following horizontal intercepts. Check your equation using graphing technology.

1. $(4, 0)$

2. $(0, 0)$ and $(4, 0)$

3. $(\text{-}2, 0)$, $(0, 0)$ and $(4, 0)$

4. $(\text{-}4, 0)$, $(0, 0)$, and $(2, 0)$

5. $(\text{-}5, 0)$, $\left(\frac{1}{2}, 0\right)$, and $(3, 0)$

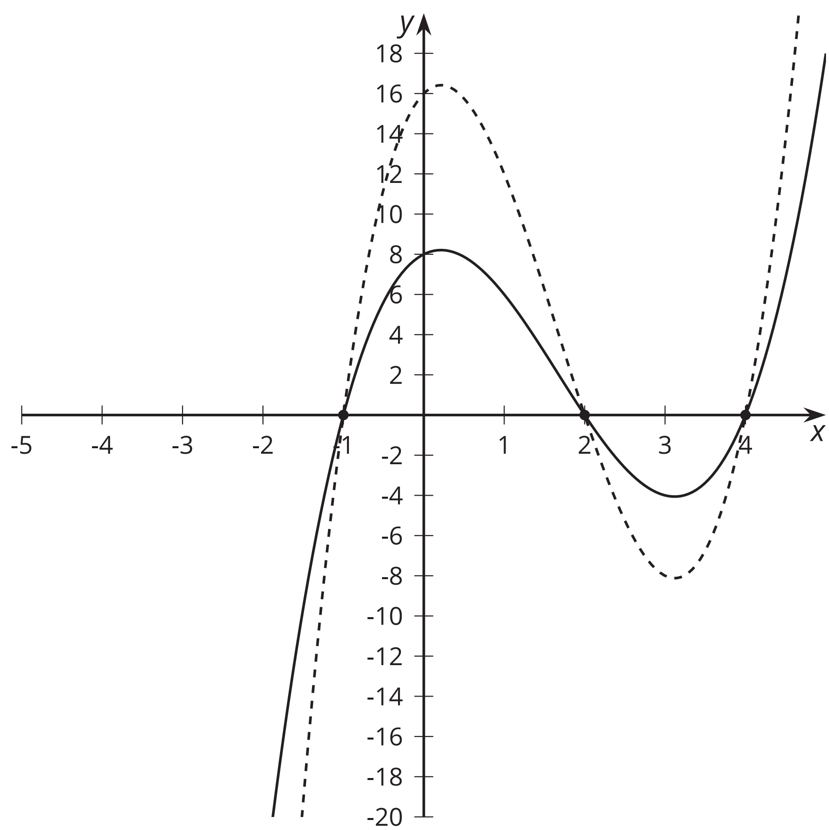
Lesson 7 Summary

We can use the zeros of a polynomial function to figure out what an expression for the polynomial might be.

Let’s say we want a polynomial function Z that satisfies $Z(x) = 0$ when x is -1, 2, or 4. We know that one way to write a polynomial expression is as a product of linear factors. We could write a possible expression for $Z(x)$ by multiplying together a factor that is zero when $x = -1$, a factor that is zero when $x = 2$, and a factor that is zero when $x = 4$. Can you think of what these three factors could be?

It turns out that there are many possible expressions for $Z(x)$. Using linear factors, one possibility is $Z(x) = (x + 1)(x - 2)(x - 4)$. Another possibility is $Z(x) = 2(x + 1)(x - 2)(x - 4)$, since the 2 (or any other rational number) does not change what values of x make the function equal to zero.

To check that these expressions match what we know about Z, we can test the three values -1, 2, and 4 to make sure that $Z(x)$ is 0 for those values. Alternatively, we can graph both possible versions of Z and see that the graphs intercept the horizontal axis at -1, 2, and 4, as shown here.

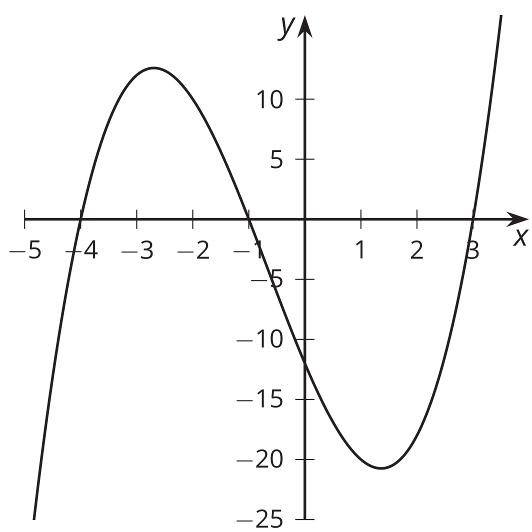


Lesson 7 Practice Problems

1. Diego wrote $f(x) = (x+2)(x-4)$ as an example of a function whose graph has x-intercepts at $x = \text{-}4, 2$. What was his mistake?

2. Write a possible equation for a polynomial whose graph has horizontal intercepts at $x = 2, \text{-}\frac{1}{2}, \text{-}3$.

3. Which polynomial function's graph is shown here?



A. $f(x) = (x+1)(x+3)(x+4)$

B. $f(x) = (x+1)(x-3)(x+4)$

C. $f(x) = (x-1)(x+3)(x-4)$

D. $f(x) = (x-1)(x-3)(x-4)$

4. Which expression is equivalent to $(3x + 2)(3x - 5)$?

A. $6x - 3$

B. $9x^2 - 10$

C. $9x^2 - 3x - 10$

D. $9x^2 - 9x - 10$

(From Unit 2, Lesson 4.)

5. What is the value of $6(x - 2)(x - 3) + 4(x - 2)(x - 5)$ when $x =$ -3?

(From Unit 2, Lesson 5.)

6. Match each polynomial function with its leading coefficient.

A. $P(x) = (x + 2)(2x - 3)(4x + 7)$	1. 40
B. $P(x) = \frac{1}{2}(x - 2)(2x - 3)(4x + 7)$	2. 8
C. $P(x) = 5(x - 2)(2x - 3)(4x + 7)$	3. 4
D. $P(x) =$ -$(x - 2)(2x - 3)(4x + 7)$	4. 2
E. $P(x) = \frac{1}{4}(x + 2)(2x - 3)(4x + 7)$	5. -8

(From Unit 2, Lesson 6.)