Lesson 4: Combining Polynomials

- Let's do arithmetic with polynomials.

4.1: Notice and Wonder: What Can Happen to Integers

What do you notice? What do you wonder?

- $7 \cdot 9 = 63$
- $7 + 9 = 16$
- $7 - 9 = -2$
- $\frac{7}{9} = 0.777\ldots$

4.2: Experimenting with Integers

Which of these statements are true? Give reasons in support of your answer.

1. If you add two even numbers, you'll always get an even number.
2. If you subtract an even number from another even number, you'll always get an even number.
3. If you add two odd numbers, you'll always get an odd number.
4. If you subtract an odd number from another odd number, you'll always get an odd number.
5. If you multiply two even numbers, you'll always get an even number.
6. If you multiply two odd numbers, you'll always get an odd number.

7. If you multiply two integers, you'll always get an integer.

8. If you add two integers, you'll always get an integer.

9. If you subtract one integer from another, you'll always get an integer.

Are you ready for more?

Which of these statements are true? Give reasons in support of your answer.

1. If you add two rational numbers, you'll always get a rational number.

2. If you multiply two rational numbers, you'll always get a rational number.

3. If you divide two rational numbers, you'll always get a rational number.

4.3: Experimenting with Polynomials

Here are some questions about polynomials. You and a partner will work on one of these questions.

1. If you add or subtract two polynomials, will you always get a polynomial?
2. If you multiply two polynomials, will you always get a polynomial?

- Try combining some polynomials to answer your question. Use the ones given by your teacher or make up your own polynomials. Keep a record of what polynomials you tried, and the results.
- When you think you have an answer to your question, explain your reasoning using equations, graphs, visuals, calculations, words, or in any way that will help others understand your reasons.

Lesson 4 Summary

If we add two integers, subtract one from the other, or multiply them, the result is another integer. The same thing is true for polynomials: combining polynomials by adding, subtracting, or multiplying will always give us another polynomial.

For example, we can multiply $-x^2 + 4.5$ and $x^3 + 2x + \sqrt{7}$ to see what happens. We'll need to use the distributive property, and there are a lot of ways to keep track of the results of distribution when we multiply polynomials. One way is to use a diagram like this:

	x^3	$2x$	$\sqrt{7}$
$-x^2$	$-x^5$	$-2x^3$	$-\sqrt{7}x^2$
4.5	$4.5x^3$	$9x$	$4.5\sqrt{7}$

Then we can find the product by adding all the results we filled in. This diagram tells us that the product is $-x^5 + 2.5x^3 - \sqrt{7}x^2 + 9x + 4.5\sqrt{7}$, which is also a polynomial even though there are square roots as coefficients! No matter what polynomials we started with, multiplying them would give us a polynomial, because we would have to multiply each part of each polynomial and then add them all together. Adding or subtracting polynomials also gives us a polynomial, because we can combine like terms.

When thinking about polynomials, it is important to remember exactly what counts as a polynomial. Any sum of terms that all have the same variable, where the variable is only raised to non-negative integer powers, is a polynomial. So some things that might not look like polynomials at first, like -34.1 or $7.9998x$, are polynomials.

Lesson 4 Practice Problems

1. Here are two expressions whose product is a new expression, A.

   $(5x^4 + \boxed{}x^3)(4x^{\boxed{}} - 6) = A$

   Andre says that any real number can go in either of the boxes and A will be a polynomial. Is he correct? Explain your reasoning.

2. Lin divides the polynomial $2x^2 - 4x + 1$ by 4 and gets $0.5x^2 - x + 0.25$. Is $0.5x^2 - x + 0.25$ a polynomial? Explain your thinking.

3. What is the result when any 2 integers are multiplied?

   A. a positive integer

   B. a negative integer

   C. an integer

   D. an even number

4. Clare wants to make an open-top box by cutting out corners of a 30 inch by 25 inch piece of poster board and then folding up the sides. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts.

   a. Write an expression for $V(x)$.

   b. What is a reasonable domain for V in this context?

   (From Unit 2, Lesson 1.)

5. Identify the degree, leading coefficient, and constant value of each of the following polynomials.

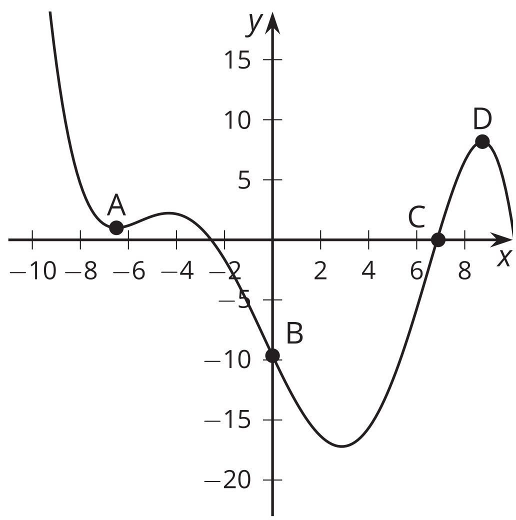
    a. $f(x) = 2x^5 - 8x^2 - x - 6$

    b. $h(x) = x^3 - 7x^2 - x + 2$

    c. $g(x) = 5x^2 - 4x^3 + 2x + 5.4$

(From Unit 2, Lesson 3.)

6. Which point is a relative minimum?



    A. A

    B. B

    C. C

    D. D

(From Unit 2, Lesson 3.)

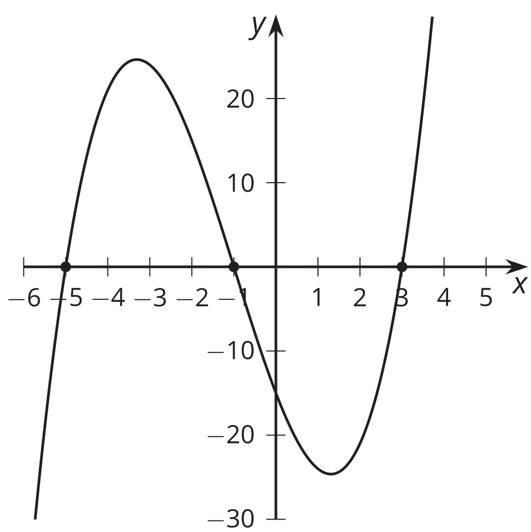
Lesson 5: Connecting Factors and Zeros

- Let's investigate polynomials written in factored form.

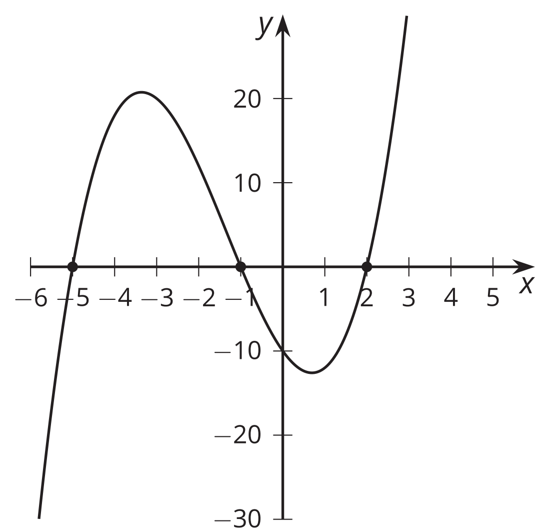
5.1: Notice and Wonder: Factored Form

What do you notice? What do you wonder?

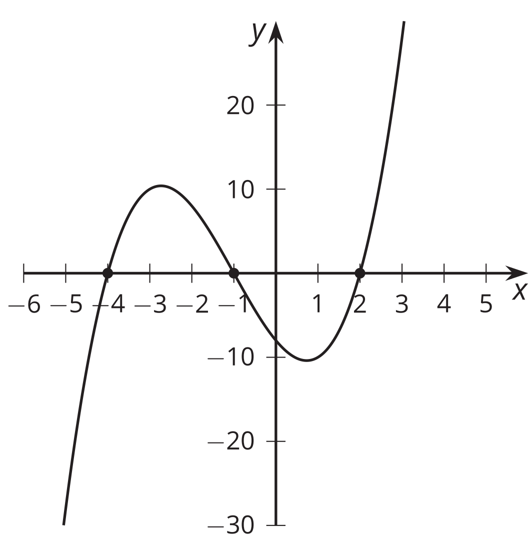
$f(x) = (x+5)(x+1)(x-3)$



$g(x) = (x+5)(x+1)(x-2)$



$h(x) = (x+4)(x+1)(x-2)$



5.2: What Values of x Make These Equations True?

Find all values of x that make the equation true.

1. $(x+4)(x+2)(x-1)=0$
2. $(x+4)(x+2)(x-1)(x-3)=0$
3. $(x+4)^2(x+2)^2=0$
4. $-2(x-4)(x-2)(x+1)(x+3)=0$
5. $(2x+8)(7x-3)(x-10)=0$
6. $x^2+3x-4=0$
7. $x(3-x)(x-1)(x+0.75)=0$
8. $(x^2-4)(x+9)=0$

Are you ready for more?

1. Write an equation that is true when x is equal to -5, 4, or 0 and for no other values of x.
2. Write an equation that is true when x is equal to -5, 4, or 0 and for no other values of x, and where one side of the equation is a 4th degree polynomial.

5.3: Factors, Intercepts, and Graphs

Your teacher will give you a set of cards. Match each equation to either a graph or a description.

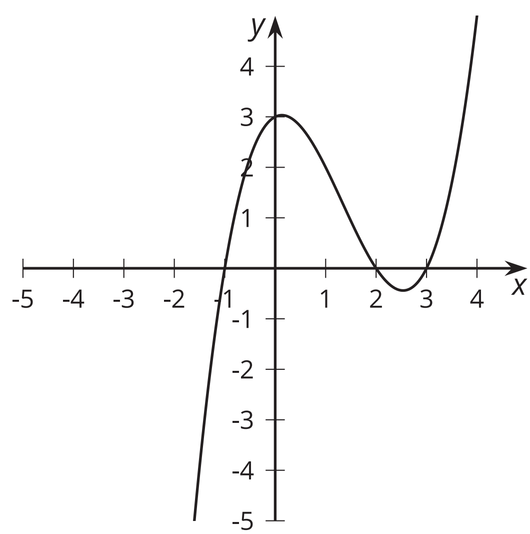
Take turns with your partner to match an equation with a graph or a description of a graph.

1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

Lesson 5 Summary

When a polynomial is written as a product of linear factors, we can identify several facts about it.

For example, the factored form of the polynomial shown in the graph is $P(x) = 0.5(x-3)(x-2)(x+1)$.



Looking back at the equation, can you see why the graph has x-intercepts at $x = 3$, 2, and -1? Each of these x-values makes one of the factors in the expression $0.5(x-3)(x-2)(x+1)$ equal to zero, and so makes the equation $P(x) = 0$ true. The numbers 3, 2, and -1 are known as the zeros of the function. When a polynomial is not written as the product of linear factors, identifying the zeros from the expression for the polynomial can be more challenging. We'll learn how to do that in future lessons.

Lesson 5 Practice Problems

1. What is the value of

$$4(x-2)(x-3)+7(x-2)(x-5)-6(x-3)(x-5)$$

   when $x=5$?

2. Which polynomial function has zeros when $x = \text{-}2, \frac{3}{4}, 5$?

   A. $f(x)=(x-2)(3x+4)(x+5)$

   B. $f(x)=(x-2)(4x+3)(x+5)$

   C. $f(x)=(x+2)(3x-4)(x+5)$

   D. $f(x)=(x+2)(4x-3)(x-5)$

3. The graph of a polynomial $f(x)=(2x-3)(x-4)(x+3)$ has x-intercepts at 3 x values. What are they?

4. Match each sequence with one of the recursive definitions. Note that only the part of the definition showing the relationship between the current term and the previous term is given so as not to give away the solutions. One of the sequences matches two recursive definitions.

   A. $a(n)=a(n-1)-4$

   B. $b(n)=b(n-1)+0$

   C. $c(n)=\text{-}\frac{1}{2}\cdot c(n-1)$

   D. $d(n)=1\cdot d(n-1)$

   1. 7, 3, -1, -5

   2. $1, \text{-}\frac{1}{2}, \frac{1}{4}, \text{-}\frac{1}{8}$

   3. 8, 8, 8, 8

   (From Unit 1, Lesson 5.)

5. Han is multiplying $10x^4$ by $0.5x^3$ and gets $5x^7$. He says that $0.5x^3$ is not a polynomial because 0.5 is not an integer. What is the error in Han's thinking? Explain your reasoning.

(From Unit 2, Lesson 4.)

6. Here are two expressions whose sum is a new expression, A.

$$(2x^2 + 5) + (6x^{\square} - 7) = A$$

Select **all** the values that we can put in the box so that A is a polynomial.

A. -2

B. -1

C. -0.5

D. 0

E. 0.5

F. 1

G. 2

(From Unit 2, Lesson 4.)