Lesson 13: Multiplying Complex Numbers

- Let's multiply complex numbers.

13.1: i Squared

Write each expression in the form $a + bi$, where a and b are real numbers.

1. $4i \cdot 3i$

2. $4i \cdot -3i$

3. $-2i \cdot -5i$

4. $-5i \cdot 5i$

5. $(-5i)^2$

13.2: Multiplying Imaginary Numbers

Take turns with your partner to match an expression in column A with an equivalent expression in column B.

- For each match that you find, explain to your partner how you know it’s a match.
- For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

A	B
$5 \cdot 7i$	-9
$5i \cdot 7i$	$35i$
$3i^2$	-35
$(3i)^2$	1
$8i^3$	9
i^4	-3
$-i^2$	-1
$(-i)^2$	$-8i$

13.3: Multiplying Complex Numbers

Write each product in the form $a + bi$, where a and b are real numbers.

1. $(-3 + 9i)(5i)$

2. $(8 + i)(-5 + 3i)$

3. $(3 + 2i)^2$

4. $(3 + 2i)(3 - 2i)$

Are you ready for more?

On October 16, 1843, while walking across the Broom Bridge in Dublin, Ireland, Sir William Rowan Hamilton came up with an idea for numbers that would work sort of like complex numbers. Instead of just the number i (and its opposite -i) squaring to give -1, he imagined three numbers i, j, and k (each with an opposite) that squared to give -1.

The way these numbers multiplied with each other was very interesting. i times j would give k, j times k would give i, and k times i would give j. But the multiplication he imagined did not have a commutative property. When those numbers were multiplied in the opposite order, they'd give the opposite number. So j times i would give -k, k times j would give -i, and i times k would give -j. A *quaternion* is a number that can be written in the form $a + bi + cj + dk$ where a, b, c, and d are real numbers.

Let $w = 2 + 3i - j$ and $z = 2i + 3k$. Write each given expression in the form $a + bi + cj + dk$.

1. $w + z$

2. wz

3. zw

Lesson 13 Summary

To multiply two complex numbers, we use the distributive property:

$$(2 + 3i)(4 + 5i) = 8 + 10i + 12i + 15i^2$$

Remember that $i^2 = \text{-}1$, so:

$$(2 + 3i)(4 + 5i) = 8 + 10i + 12i - 15$$

When we add the real parts together and the imaginary parts together, we get:

$$(2 + 3i)(4 + 5i) = \text{-}7 + 22i$$

Lesson 13 Practice Problems

1. Which expression is equivalent to $2i(5 + 3i)$?

   A. $-6 + 10i$

   B. $6 + 10i$

   C. $-10 + 6i$

   D. $10 + 6i$

2. Lin says, “When you add or multiply two complex numbers, you will always get an answer you can write in $a + bi$ form.”

   Noah says, “I don’t think so. Here are some exceptions I found:”

   $(7 + 2i) + (3 - 2i) = 10$

   $(2 + 2i)(2 + 2i) = 8i$

   a. Check Noah’s arithmetic. Is it correct?

   b. Can Noah’s answers be written in the form $a + bi$, where a and b are real numbers? Explain or show your reasoning.

3. Explain to someone who missed class how you would write $(3 - 5i)(-2 + 4i)$ in the form $a + bi$, where a and b are real numbers.

4. Which expression is equal to $729^{\frac{2}{3}}$?

A. 243

B. 486

C. 9^2

D. 27^3

(From Unit 3, Lesson 4.)

5. Find the solution(s) to each equation, or explain why there is no solution.

a. $2x^2 - \frac{2}{3} = 5\frac{1}{3}$

b. $(x + 1)^2 = 81$

c. $3x^2 + 14 = 12$

(From Unit 3, Lesson 7.)

6. Plot each number in the complex plane.

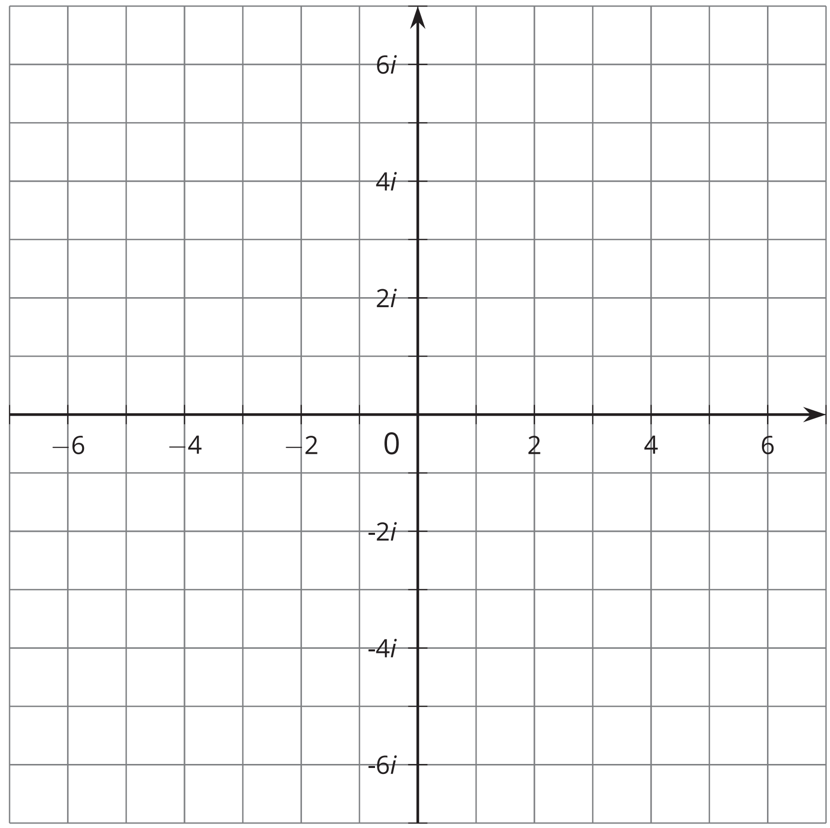
a. $5i$

b. $2 + 4i$

c. -3

d. $1 - 3i$

e. -5 $- 2i$



(From Unit 3, Lesson 11.)

7. Select **all** the expressions that are equivalent to $(3x + 2)(x - 4)$ for all real values of x.

A. $3x^2 - 12$

B. $3x^2 - 10x - 8$

C. $3(x^2 + 2x - 4)$

D. $3(x^2 - 3x) - (x + 8)$

E. $3x(x - 3) - 2(5x + 4)$

F. $3x(x - 4) + 2(x - 4)$

(From Unit 2, Lesson 23.)

Lesson 14: More Arithmetic with Complex Numbers

- Let's practice adding, subtracting, and multiplying complex numbers.

14.1: Which One Doesn't Belong: Complex Expressions

Which one doesn't belong?

A. i^2

B. $(1+i)+(1-i)$

C. $(1+i)^2$

D. $(1+i)(1-i)$

14.2: Powers of i

1. Write each power of i in the form $a+bi$, where a and b are real numbers. If a or b is zero, you can ignore that part of the number. For example, $0+3i$ can simply be expressed as $3i$.

   i^0

   i^1

   i^2

   i^3

   i^4

   i^5

   i^6

   i^7

   i^8

2. What is i^{100}? Explain your reasoning.

3. What is i^{38}? Explain your reasoning.

Are you ready for more?

1. Write each power of $1 + i$ in the form $a + bi$, where a and b are real numbers. If a or b is zero, you can ignore that part of the number. For example, $0 + 3i$ can simply be expressed as $3i$.

   a. $(1 + i)^0$

   b. $(1 + i)^1$

   c. $(1 + i)^2$

   d. $(1 + i)^3$

   e. $(1 + i)^4$

   f. $(1 + i)^5$

   g. $(1 + i)^6$

   h. $(1 + i)^7$

   i. $(1 + i)^8$

2. Compare and contrast the powers of $1 + i$ with the powers of i. What is the same? What is different?

14.3: Add 'Em Up (or Subtract or Multiply)

For each row, your partner and you will each rewrite an expression so it has the form $a + bi$, where a and b are real numbers. You and your partner should get the same answer. If you disagree, work to reach agreement.

partner A	partner B
$(7 + 9i) + (3 - 4i)$	$5i(1 - 2i)$
$2i(3 + 4i)$	$(1 + 2i) - (9 - 4i)$
$(4 - 3i)(4 + 3i)$	$(5 + i) + (20 - i)$
$(2i)^4$	$(3 + i\sqrt{7})(3 - i\sqrt{7})$
$(1 + i\sqrt{5}) - (\text{-}7 - i\sqrt{5})$	$(\text{-}2i)(\text{-}\sqrt{5} + 4i)$
$\left(\frac{1}{2}i\right)\left(\frac{1}{3}i\right)\left(\frac{3}{4}i\right)$	$\left(\frac{1}{2}i\right)^3$

Lesson 14 Summary

Suppose we want to write the product $(3+5i)(7-2i)$ in the form $a+bi$, where a and b are real numbers. For example, we might want to compare our solution with a partner's, and having answers in the same form makes that easier. Using the distributive property,

$$\begin{aligned}(3+5i)(7-2i) &= 21-6i+35i-10i^2\\ &= 21+29i-10(\text{-}1)\\ &= 21+29i+10\\ &= 31+29i\end{aligned}$$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that $i^2 = \text{-}1$ with the fact that $\text{-}i^2 = \text{-}(\text{-}1) = 1$.

Next, suppose we want to write the difference $(\text{-}6+3i)-(2-4i)$ as a single complex number in the form $a+bi$. Distributing the negative and combining like terms, we get:

$$\begin{aligned}(\text{-}6+3i)-(2-4i) &= \text{-}6+3i-2-(\text{-}4i)\\ &= \text{-}8+3i+4i\\ &= \text{-}8+7i\end{aligned}$$

Again, it is important to be precise with negative signs. It is a common mistake to just subtract $4i$ rather than subtracting $\text{-}4i$.

Lesson 14 Practice Problems

1. Select **all** expressions that are equivalent to $8 + 16i$.

   A. $2(4 + 8i)$

   B. $2i(8 - 4i)$

   C. $4(2i - 4)$

   D. $4i(4 - 2i)$

   E. $-2i(-8 - 4i)$

2. Which expression is equivalent to $(-4 + 3i)(2 - 7i)$?

   A. $-29 - 22i$

   B. $-29 + 34i$

   C. $13 - 22i$

   D. $13 + 34i$

3. Match the equivalent expressions.

   A. $i^2(3 + i)$

   B. $-4i \cdot 5i$

   C. $5i(4 - 3i)$

   D. $(1 + 2i)(-1 + 3i)$

   1. $(3 + 5i) - (10 + 4i)$

   2. $(2 + 4i)(2 - 4i)$

   3. $(1 - 4i) + (-4 + 3i)$

   4. $(-6 + 12i) - (-21 - 8i)$

4. Write each expression in $a + bi$ form.

   a. $(-8 + 3i) - (2 + 5i)$

   b. $7i(4 - i)$

   c. $(3i)^3$

   d. $(3 + 5i)(4 + 3i)$

   e. $(3i)(-2i)(4i)$

5. Here is a method for solving the equation $\sqrt{5 + x} + 10 = 6$. Does the method produce the correct solution to the equation? Explain how you know.

$$\begin{aligned} \sqrt{5+x} + 10 &= 6 & \\ \sqrt{5+x} &= -4 & \text{(after subtracting 10 from each side)} \\ 5 + x &= 16 & \text{(after squaring both sides)} \\ x &= 11 & \end{aligned}$$

(From Unit 3, Lesson 7.)

6. Write each expression in the form $a + bi$, where a and b are real numbers.

   a. $4(3 - i)$

   b. $(4 + 2i) + (8 - 2i)$

   c. $(1 + 3i)(4 + i)$

   d. $i(3 + 5i)$

   e. $2i \cdot 7i$

(From Unit 3, Lesson 13.)