Lesson 10: Interpreting and Writing Logarithmic Equations

- Let's look at logarithms with different bases.

10.1: Reading Logs

The expression $\log_{10} 1{,}000 = 3$ can be read as: "The log, base 10, of 1,000 is 3."

It can be interpreted as: "The exponent to which we raise a base 10 to get 1,000 is 3."

Take turns with a partner reading each equation out loud. Then, interpret what they mean.

- $\log_{10} 100{,}000{,}000 = 8$
- $\log_{10} 1 = 0$
- $\log_{2} 16 = 4$
- $\log_{5} 25 = 2$

10.2: Base 2 Logarithms

x	$\log_2(x)$
1	0
2	1
3	1.5850
4	2
5	2.3219
6	2.5850
7	2.8074
8	3
9	3.1699
10	3.3219

x	$\log_2(x)$
11	3.4594
12	3.5845
13	3.7004
14	3.8074
15	3.9069
16	4
17	4.0875
18	4.1699
19	4.2479
20	4.3219

x	$\log_2(x)$
21	4.3923
22	4.4594
23	4.5236
24	4.5850
25	4.6439
26	4.7004
27	4.7549
28	4.8074
29	4.8580
30	4.9069

x	$\log_2(x)$
31	4.9542
32	5
33	5.0444
34	5.0875
35	5.1293
36	5.1699
37	5.2095
38	5.2479
39	5.2854
40	5.3219

1. Use the table to find the exact or approximate value of each expression. Then, explain to a partner what each expression and its approximated value means.
    a. $\log_2 2$
    b. $\log_2 32$
    c. $\log_2 15$
    d. $\log_2 40$
2. Solve each equation. Write the solution as a logarithmic expression.
    a. $2^y = 5$
    b. $2^y = 70$
    c. $2^y = 999$

10.3: Exponential and Logarithmic Forms

These equations express the same relationship between 2, 16, and 4:

$$\log_2 16 = 4 \qquad\qquad 2^4 = 16$$

1. Each row shows two equations that express the same relationship. Complete the table.

	exponential form	logarithmic form
a.	$2^1 = 2$	
b.	$10^0 = 1$	
c.		$\log_3 81 = 4$
d.		$\log_5 1 = 0$
e.	$10^{-1} = \frac{1}{10}$	
f.	$9^{\frac{1}{2}} = 3$	
g.		$\log_2 \frac{1}{8} = -3$
h.	$2^y = 15$	
i.		$\log_5 40 = y$
j.	$b^y = x$	

2. Write two equations—one in exponential form and one in logarithmic form—to represent each question. Use "?" for the unknown value.
    a. "To what exponent do we raise the number 4 to get 64?"

    b. "What is the log, base 2, of 128?"

Are you ready for more?

1. Is $\log_2(10)$ greater than 3 or less than 3? Is $\log_{10}(2)$ greater than or less than 1? Explain your reasoning.

2. How are these two quantities related?

Lesson 10 Summary

Many relationships that can be expressed with an exponent can also be expressed with a logarithm. Let's look at this equation:

$$2^7 = 128$$

The base is 2 and the exponent is 7, so it can be expressed as a logarithm with base 2:

$$\log_2 128 = 7$$

In general, an exponential equation and a logarithmic equation are related as shown here:

Exponents can be negative, so a logarithm can have negative values. For example $3^{-4} = \frac{1}{81}$, which means that $\log_3 \frac{1}{81} = -4$.

An exponential equation cannot always be solved by observation. For example, $2^x = 19$ does not have an obvious solution. The logarithm gives us a way to represent the solution to this equation: $x = \log_2 19$. The expression $\log_2 19$ is approximately 4.25, but $\log_2 19$ is an exact solution.

Lesson 10 Practice Problems

1. a. Use the base-2 log table (printed in the lesson) to approximate the value of each exponential expression.
   i. 2^5
   ii. $2^{3.7}$
   iii. $2^{4.25}$

   b. Use the base-2 log table to find or approximate the value of each logarithm.
   i. $\log_2 4$
   ii. $\log_2 17$
   iii. $\log_2 35$

2. Here is a logarithmic expression: $\log_2 64$.

   a. How do we say the expression in words?

   b. Explain in your own words what the expression means.

   c. What is the value of this expression?

3. a. What is $\log_{10}(100)$? What about $\log_{100}(10)$?

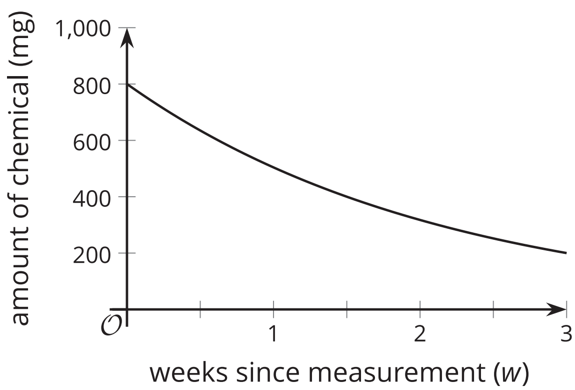
   b. What is $\log_2(4)$? What about $\log_4(2)$?

   c. Express b as a power of a if $a^2 = b$.

4. In order for an investment, which is increasing in value exponentially, to increase by a factor of 5 in 20 years, about what percent does it need to grow each year? Explain how you know.

(From Unit 4, Lesson 4.)

5. Here is the graph of the amount of a chemical remaining after it was first measured. The chemical decays exponentially.



What is the approximate half-life of the chemical? Explain how you know.

(From Unit 4, Lesson 7.)

6. Find each missing exponent.

a. $10^{?} = 100$

b. $10^{?} = 0.01$

c. $\left(\frac{1}{10}\right)^{?} = \frac{1}{1{,}000}$

d. $2^{?} = \frac{1}{2}$

e. $\left(\frac{1}{2}\right)^{?} = 2$

(From Unit 4, Lesson 8.)

7. Explain why $\log_{10} 1 = 0$.

(From Unit 4, Lesson 9.)

8. How are the two equations $10^2 = 100$ and $\log_{10}(100) = 2$ related?

(From Unit 4, Lesson 9.)

Lesson 11: Evaluating Logarithmic Expressions

- Let's find some logs!

11.1: Math Talk: Finding Values

Evaluate mentally.

1. $\log 10$
2. $\log 10{,}000$
3. $\log 0.1$
4. $\log \frac{1}{1{,}000}$

11.2: Log War!

Have you played the game of war with a deck of playing cards?

Your teacher will give you and your partner a set of special cards.

- Shuffle and deal the cards evenly.
- Each player turns one card face up. The card with the greater value wins the round and its player captures both cards and sets them aside.
- If you and your partner disagree about the value of a card, discuss until you reach an agreement.
- If there is a tie, each player turns another card face up. The player whose card has the greater value captures all cards (including the cards that tie).
- Play until all the cards are turned up. The player with the most cards wins.

Let the logarithm war begin!

Are you ready for more?

Mai uses the fact that $\sqrt[3]{2}$ is close to 1.25 and that $2^3 = 8$ to make the estimate $\log_2(10) \approx 3\frac{1}{3}$. Explain Mai's estimate. How exact is the estimate?

11.3: Finding Logarithms with a Calculator

1. To solve the equation $10^m = 19$, Tyler writes the equation in the logarithmic form: $m = \log_{10} 19$. He then presses the "log" button on the calculator, enters the number "19," and writes down an approximation of 1.279. Priya follows the same steps on her calculator and writes down 1.27875.
   a. Experiment with your calculator until you understand how to evaluate $\log_{10} 19$. What value do you see on the calculator?

   b. Discuss with a partner: Why might $\log_{10} 19$ be expressed in different ways?

2. Express the solution to each equation using a logarithm. Next, find the approximate value of the solution using a calculator.
   a. $10^m = 24$

   b. $10^n = 750$

3. Estimate the value of each expression. Explain to a partner how you made your estimate. Next, check your estimate with a calculator.
   a. $\log 90$

   b. $\log 1{,}005$

   c. $\log 9$

Lesson 11 Summary

Sometimes it’s possible to find an exact value for a logarithm. For example $\log_2 0.125 = -3$ because $0.125 = \frac{1}{8}$ and $2^{-3} = \frac{1}{8}$. Similarly, $\log_5 625 = 4$ because $5^4 = 625$.

Often times it is not possible to find an exact value, but using number sense allows us to get a reasonable estimate. Let’s say we want to find x that makes $10^x = 980$ true.

We can first express x as $\log_{10} 980$. Because 980 is between 10^2 and 10^3, $\log_{10} 980$ is between 2 and 3.

But where does $\log_{10} 980$ lie between 2 and 3? Because 980 is much closer to 1,000 than it is to 100, $\log_{10} 980$ is likely a lot closer to 3 than it is to 2. This means 2.9 would be a better estimate than 2.1 would be.

We can use a calculator to verify our estimate and find that $\log_{10} 980$ is *very* close to 3, about 2.99.

Lesson 11 Practice Problems

1. Select **all** expressions that are equal to $\log_2 8$.

    A. $\log_5 20$

    B. $\log_5 125$

    C. $\log_{10} 100$

    D. $\log_{10} 1{,}000$

    E. $\log_3 27$

    F. $\log_{10} 0.001$

2. Which expression has a greater value: $\log_{10} \frac{1}{100}$ or $\log_2 \frac{1}{8}$? Explain how you know.

3. Andre says that $\log_{10}(55) = 1.5$ because 55 is halfway between 10 and 100. Do you agree with Andre? Explain your reasoning.

4. An exponential function is defined by $k(x) = 15 \cdot 2^x$.

    a. Show that when x increases from 1 to 1.25 and when it increases from 2.75 to 3, the value of k grows by the same factor.

    b. Show that when x increases from t to $t + 0.25$, $k(t)$ also grows by this same factor.

(From Unit 4, Lesson 5.)

5. How many times does $1 need to double in value to become $1,000,000? Explain how you know.

(From Unit 4, Lesson 8.)

6. What values could replace the "?" in these equations to make them true?

    a. $\log_{10} 10{,}000 = ?$

    b. $\log_{10} 10{,}000{,}000 = ?$

    c. $\log_{10} ? = 5$

    d. $\log_{10} ? = 1$

(From Unit 4, Lesson 9.)

7. a. What value of t would make the equation $2^t = 6$ true?

b. Between which two whole numbers is the value of $\log_2 6$? Explain how you know.

(From Unit 4, Lesson 10.)

8. For each exponential equation, write an equivalent equation in logarithmic form.

a. $3^4 = 81$

b. $10^0 = 1$

c. $4^{\frac{1}{2}} = 2$

d. $2^t = 5$

e. $m^n = C$

(From Unit 4, Lesson 10.)