Complex Numbers and Rational Exponents: End-of-Unit Assessment

Do not use a calculator.

1. Select **all** expressions that are equivalent to $64^{\frac{2}{3}}$.

   A. $\left(\sqrt{64}\right)^3$

   B. $\left(\sqrt[3]{64}\right)^2$

   C. 4^2

   D. $\sqrt[3]{64^2}$

   E. $\sqrt[3]{128}$

2. How many real solutions does $x^2 + 8x + 20 = 0$ have?

   A. 0

   B. 1

   C. 2

3. Select **all** the solutions to $(x-2)^2 = \text{-}16$.

   A. $x = 6$

   B. $x = \text{-}2$

   C. $x = \text{-}6$

   D. $x = 2 + 4i$

   E. $x = 2 + 2i$

   F. $x = 2 - 2i$

   G. $x = 2 - 4i$

4. Let $p = 5 - 2i$ and $q = \text{-}3 + 7i$. Write each expression in the form $a + bi$:

a. $p + q$

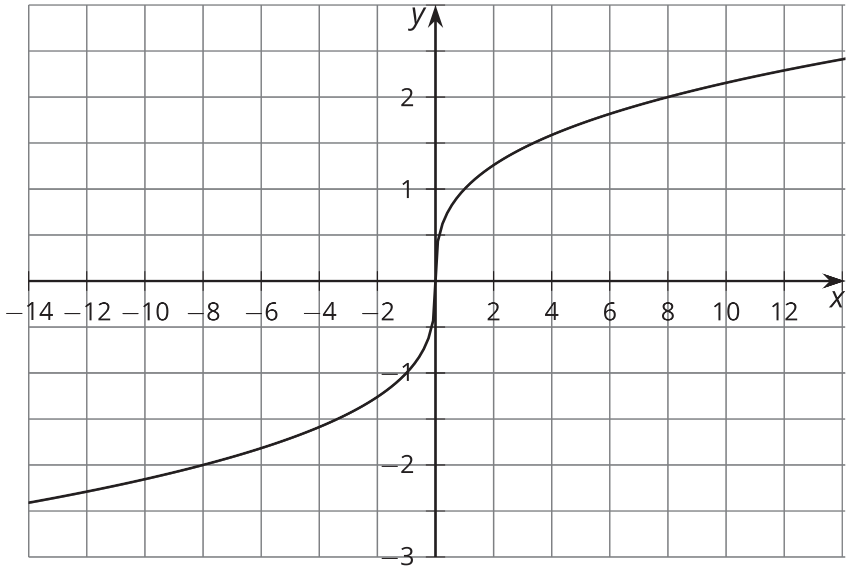
b. $p - q$

c. pq

5. a. Show how to solve the equation $\sqrt{2x + 1} - 4 = \text{-}1$.

b. Explain why $\sqrt{2x + 1} + 4 = \text{-}1$ has no real solution.

6. a. Here is a graph of $g(x) = \sqrt[3]{x}$.



Use the graph of $g(x) = \sqrt[3]{x}$ to help you explain why there is only one x-intercept for every cube root function of the form $y = \sqrt[3]{x + a}$, in which a is a real number.

b. Use the meaning of cube roots to show how to find an exact solution to the equation $\sqrt[3]{x + 2} = -2$ without using a graph.

c. Use the meaning of cube roots to show how to find an exact solution to the equation $\sqrt[3]{x} + 2 = -2$ without using a graph.

7. Noah and Lin are each trying to solve the equation $x^2 - 6x + 10 = 0$. They know that the solutions to $x^2 = -1$ are i and $-i$, but they are not sure how to use this information to solve for x in their equation.

   a. Here is Noah's work:

   $$x^2 - 6x + 10 = 0$$
   $$x^2 - 6x = -10$$
   $$x^2 - 6x + 9 = -10 + 9$$
   $$(x-3)^2 = -1$$

   Show how Noah can finish his work using complex numbers.

   b. Lin decides to solve the equation using the quadratic formula. Here is her work:

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
   $$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$
   $$x = \frac{6 \pm \sqrt{36 - 40}}{2}$$

   Lin knows $36 - 40$ is a negative number and isn't sure what to do next. Show how Lin can write her solution using i.