Lesson 15: Working Backwards

Let's use what we've learned about multiplying complex numbers.

15.1: What's Missing?

Here are some complex numbers with an unknown difference: $(10+4i)-(\_\_+\_\_i)=?$

1. If the result of this subtraction is a real number, what could the second complex number be?
2. If the result of this subtraction is an imaginary number, what could the second complex number be?

15.2: Info Gap: What Was Multiplied?

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Lesson 15 Summary

When complex numbers are multiplied, each part of one of the numbers gets distributed to the other one. This means that we'll always see the same pattern:

$$(a + bi)(c + di) = ac + adi + bci + bdi^2$$

We can use the fact that $i^2 = -1$ to rearrange this and make it easier to see the real part and the imaginary part of the result.

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

Every time we multiply complex numbers, the result is not only a complex number, but it's a specific complex number that comes from combining the parts of the numbers we started with in a specific way. If a and c are the real parts of the numbers we start with and bi and di are the imaginary parts, then the result will always have $ac - bd$ as a real part and $(ad + bc)i$ as an imaginary part.

Lesson 15 Practice Problems

1. Select **all** the expressions that are equivalent to $(3 - 5i)(-8 + 2i)$.

   A. $-24 + 6i - 40i + 10i^2$

   B. $-24 + 46i - 10$

   C. $-24 + 6i + 40i - 10i^2$

   D. $-14 - 34i$

   E. $-34 - 34i$

   F. $-24 + 46i + 10$

   G. $46i - 14$

   H. $-34 + 46i$

2. Explain or show how to write $(20 - i)(8 + 4i)$ in the form $a + bi$, where a and b are real numbers.

3. Without going through all the trouble of writing the left side in the form $a + bi$, how could you tell that this equation is false?

   $(-9 + 2i)(10 - 13i) = -68 - 97i$

4. Andre spilled something on his math notebook and some parts of the problems he was working on were erased. Here is one of the problems:

$$(\quad -2i)(\quad +2i) = \quad -10i$$

   a. What could go in the blanks?

   b. Could other numbers work, or is this the only possibility? Explain your reasoning.

5. Find the exact solution(s) to each of these equations, or explain why there is no solution.

   a. $x^2 = 49$

   b. $x^3 = 49$

   c. $x^2 = -49$

   d. $x^3 = -49$

(From Unit 3, Lesson 8.)

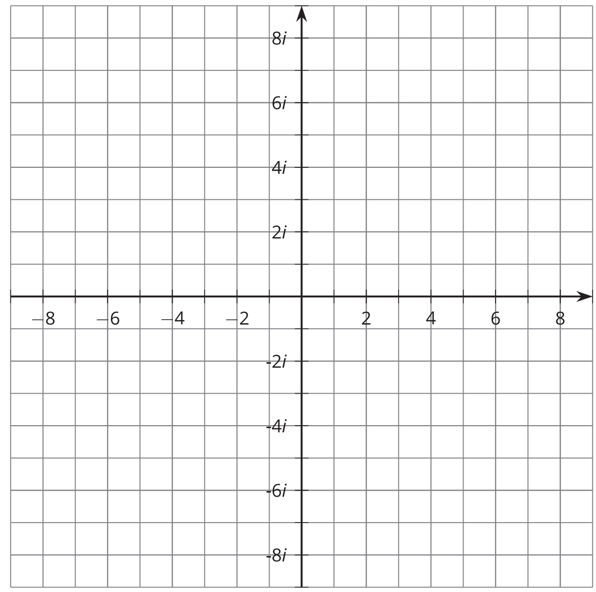
6. Write each expression in the form $a + bi$, where a and b are real numbers. Optionally, plot $3 + 2i$ in the complex plane. Then plot and label each of your answers.

   a. $2(3 + 2i)$

   b. $i(3 + 2i)$

   c. $-i(3 + 2i)$

   d. $(3 - 2i)(3 + 2i)$



(From Unit 3, Lesson 13.)

7. The table shows two investment account balances growing over time.

time (years since 2000)	account A (thousands of dollars)	account B (thousands of dollars)
0	5	10
1	5.1	10.15
2	5.2	10.3
3	5.3	10.45
4	5.4	10.6

a. Describe a pattern in how each account balance changed from one year to the next.

b. Define the amount of money, in thousands of dollars, in accounts A and B as functions of time t, where t is years since 2000, using function notation.

c. Will account A ever have the same balance as account B? If so, when? Explain how you know.

(From Unit 1, Lesson 10.)

Lesson 16: Solving Quadratics

- Let's solve quadratic equations.

16.1: Find the Perfect Squares

The expression $x^2 + 8x + 16$ is equivalent to $(x + 4)^2$. Which expressions are equivalent to $(x + n)^2$ for some number n?

1. $x^2 + 10x + 25$

2. $x^2 + 10x + 29$

3. $x^2 - 6x + 8$

4. $x^2 - 6x + 9$

16.2: Different Ways to Solve It

Elena and Han solved the equation $x^2 - 6x + 7 = 0$ in different ways.

Elena said, "First I added 2 to each side:

$$x^2 - 6x + 7 + 2 = 2$$

So that tells me:

$$(x - 3)^2 = 2$$

I can find the square roots of both sides:

$$x - 3 = \pm\sqrt{2}$$

Which is the same as:

$$x = 3 \pm \sqrt{2}$$

So the two solutions are $x = 3 + \sqrt{2}$ and $x = 3 - \sqrt{2}$."

Han said, “I used the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Since $x^2 - 6x + 7 = 0$, that means $a = 1$, $b = -6$, and $c = 7$. I know:

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

or

$$x = \frac{6 \pm \sqrt{8}}{2}$$

So:

$$x = 3 \pm \frac{\sqrt{8}}{2}$$

I think the solutions are $x = 3 + \frac{\sqrt{8}}{2}$ and $x = 3 - \frac{\sqrt{8}}{2}$.”

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

Under what circumstances would solving an equation of the form $x^2 + bx + c = 0$ lead to a solution that doesn’t involve fractions?

16.3: Solve These Ones

Solve each quadratic equation with the method of your choice. Be prepared to compare your approach with a partner's.

1. $x^2 = 100$

2. $x^2 = 38$

3. $x^2 - 10x + 25 = 0$

4. $x^2 + 14x + 40 = 0$

5. $x^2 + 14x + 39 = 0$

6. $3x^2 - 5x - 11 = 0$

Lesson 16 Summary

Consider the quadratic equation:

$$x^2 - 5x = 25$$

It is often most efficient to solve equations like this by completing the square. To complete the square, note that the perfect square $(x+n)^2$ is equal to $x^2 + (2n)x + n^2$. Compare the coefficients of x in $x^2 + (2n)x + n^2$ to our expression $x^2 - 5x$ to see that we want $2n = -5$, or just $n = -\frac{5}{2}$. This means the perfect square $\left(x - \frac{5}{2}\right)^2$ is equal to $x^2 - 5x + \frac{25}{4}$, so adding $\frac{25}{4}$ to each side of our equation will give us a perfect square.

$$\begin{aligned} x^2 - 5x &= 25 \\ x^2 - 5x + \frac{25}{4} &= 25 + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{100}{4} + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{125}{4} \end{aligned}$$

The two numbers that square to make $\frac{125}{4}$ are $\frac{\sqrt{125}}{2}$ and $-\frac{\sqrt{125}}{2}$, so:

$$x - \frac{5}{2} = \pm\frac{\sqrt{125}}{2}$$

which means the two solutions are:

$$x = \frac{5}{2} \pm \frac{\sqrt{125}}{2}$$

Other times, it is most efficient to use the quadratic formula. Look at the quadratic equation:

$$3x^2 - 2x = 0.8$$

We could divide each side by 3 and then complete the square like before, but the equation would get even messier and the chance of making a mistake might be higher. With messier equations like this, it is often most efficient to use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use this formula, we first need to put the equation in standard form and identify a, b, and c. Rearranging, we get:

$$3x^2 - 2x - 0.8 = 0$$

so $a = 3$, $b = \text{-}2$, and $c = \text{-}0.8$. We have to be careful to pay attention to the negative signs. Using the quadratic formula, we get:

$$x = \frac{\text{-}(\text{-}2) \pm \sqrt{(\text{-}2)^2 - 4(3)(\text{-}0.8)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + (12)(0.8)}}{6}$$

Evaluating these solutions with a calculator gives decimal approximations -0.281 and 0.948.

Lesson 16 Practice Problems

1. What number should be added to the expression $x^2 - 15x$ to result in an expression equivalent to a perfect square?

   A. -7.5

   B. 7.5

   C. -56.25

   D. 56.25

2. Noah uses the quadratic formula to solve the equation $2x^2 + 3x - 5 = 4$. He finds $x =$ -2.5 or 1. But, when he checks his answer, he finds that neither -2.5 nor 1 are solutions to the equation. Here are his steps:

   $a = 2$, $b = 3$, $c =$ -5

   $x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2}$

   $x = \frac{-3 \pm \sqrt{49}}{4}$

   $x =$ -2.5 or 1

   a. Explain what Noah's mistake was.

   b. Solve the equation correctly.

3. Solve each quadratic equation with the method of your choice.

a. $x^2 - 2x = -1$

b. $x^2 + 8x + 14 = 23$

c. $x^2 - 15 = 0$

d. $7x^2 - 2x - 5 = 0$

e. $2x^2 + 12x = 8$

4. What are the solutions to the equation $x^2 - 4x = -3$?

A. $\frac{4 \pm \sqrt{16 - 4 \cdot 0 \cdot -3}}{2 \cdot 0}$

B. $\frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot -3}}{2 \cdot 1}$

C. $\frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$

D. $\frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$

5. Which expression is equivalent to $\sqrt{-23}$?

A. $-23i$

B. $23i$

C. $-i\sqrt{23}$

D. $i\sqrt{23}$

(From Unit 3, Lesson 11.)

6. Write each expression in the form $a + bi$, where a and b are real numbers.

a. $5i^2$

b. $i^2 \cdot i^2$

c. $(-3i)^2$

d. $7 \cdot 4i$

e. $(5 + 4i) - (-3 + 2i)$

(From Unit 3, Lesson 12.)

7. Let $m = (7 - 2i)$ and $k = 3i$. Write each expression in the form $a + bi$, where a and b are real numbers.

a. $k - m$

b. k^2

c. m^2

d. $k \cdot m$

(From Unit 3, Lesson 13.)