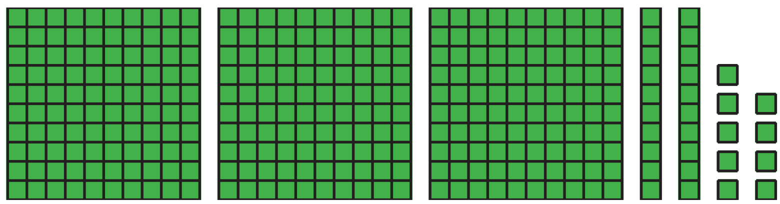
Lesson 2: Funding the Future

- Let’s look at some other things that polynomials can model.

2.1: Notice and Wonder: Writing Numbers

What do you notice? What do you wonder?



$$300 + 20 + 9$$

3 100s, 2 10s, 9 1s

$$3(10^2) + 2(10^1) + 9(10^0)$$

2.2: Polynomials in the Integers

Consider the polynomial function p given by $p(x) = 5x^3 + 6x^2 + 4x$.

1. Evaluate the function at $x =$ -5 and $x = 15$.

2. How does knowing that $5{,}000 + 600 + 40 = 5{,}640$ help you solve the equation $5x^3 + 6x^2 + 4x = 5{,}640$?

Are you ready for more?

Han notices:

- $11^2 = 121$ and $(x+1)^2 = x^2 + 2x + 1$
- $11^3 = 1331$ while $(x+1)^3 = x^3 + 3x^2 + 3x + 1$

The digits in the powers of 11 correspond to the coefficients of the polynomials.

1. Is this still true for 11^4 and $(x+1)^4$? What about 11^5 and $(x+1)^5$?

2. Give a mathematical justification of Han's observation.

2.3: A Yearly Gift

At the end of 12th grade, Clare's aunt started investing money for her to use after graduating from college four years later. The first deposit was $300. If r is the annual interest rate of the account, then at the end of each school year the balance in the account is multiplied by a growth factor of $x = 1 + r$.

1. After one year, the total value is $300x$. After two years, the total value is $300x \cdot x = 300x^2$. Write an expression for the total value after graduation in terms of x.

2. If Clare's aunt had invested another $500 at the end of her freshman year, what would the expression be for the total value after graduation in terms of x?

   Pause here for a whole-class discussion.

3. Suppose that $250 was invested at the end of sophomore year, and $400 at the end of junior year in addition to the original $300 and the $500 invested at the end of freshman year. Write an expression for the total value after graduation in terms of x.

4. The total amount y, in dollars, after four years is a function $y = C(x)$ of the growth factor x. If the total Clare receives after graduation is $C(x) = 1{,}580$, use a graph to find the interest rate that the account earned.

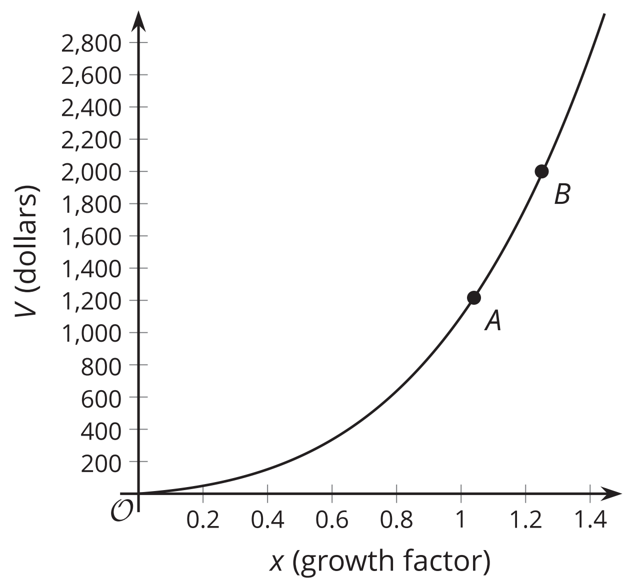
Lesson 2 Summary

Let's say we're going to invest \$200 at an annual interest rate of r. This means at the end of a year, the balance in the account is multiplied by a growth factor of $x = 1 + r$. After the first year, the amount in the account can be expressed as $200x$, which is a polynomial. Similarly, after the second year, the amount will be $200x^2$, after three years, the amount will be $200x^3$, etc.

If an additional \$350 is invested at the end of the first year, we can revise the polynomial. The amount of money in the account after 1 year is the same, but now the amount of money after two years is $(200x + 350)x = 200x^2 + 350x$.

What will the polynomial expression look like if \$400 more is invested at the end of the second year and \$150 more is invested at the end of the third year?
$200x^4 + 350x^3 + 400x^2 + 150x$.

Let $D(x)$ be the amount of money in dollars in the account after four years and x be the growth factor where $D(x) = 200x^4 + 350x^3 + 400x^2 + 150x$. A graph of $y = D(x)$ helps us visualize how the amount in the account after four years depends on different values of x.



We can use this polynomial model to examine the effect of different annual interest rates, or to estimate what the annual interest rate needs to be to achieve a specific quantity at the end of the four years. For example, point A is at $(1.04, D(1.04)) \approx (1.04, 1216)$. From this, we know that the amount in the account after 4 years with an interest rate of 4% each year is approximately \$1,216. Similarly, if we want the account to have \$2,000 after four years, that corresponds to point B, and at that point the growth rate is approximately 1.25 each year, since $(1.25, D(1.25)) \approx (1.25, 2000)$. So an interest rate of 25% will get us there, though we are not likely to find a bank that would offer that rate. Note also that the values $x < 1$ correspond to negative rates, which are also unlikely!

Polynomial models are adaptable to a variety of situations even as they grow in complexity.

Lesson 2 Practice Problems

1. Select **all** polynomial expressions that are equivalent to $6x^4 + 4x^3 - 7x^2 + 5x + 8$.

   A. $16x^{10}$

   B. $6x^5 + 4x^4 - 7x^3 + 5x^2 + 8x$

   C. $6x^4 + 4x^3 - 7x^2 + 5x + 8$

   D. $8 + 5x + 7x^2 - 4x^3 + 6x^4$

   E. $8 + 5x - 7x^2 + 4x^3 + 6x^4$

2. Each year a certain amount of money is deposited in an account which pays an annual interest rate of r so that at the end of each year the balance in the account is multiplied by a growth factor of $x = 1 + r$. $500 is deposited at the start of the first year, an additional $200 is deposited at the start of the next year, and $600 at the start of the following year.

   a. Write an expression for the value of the account at the end of three years in terms of the growth factor x.

   b. What is the amount (to the nearest cent) in the account at the end of three years if the interest rate is 2%?

3. Consider the polynomial function p given by $p(x) = 5x^3 + 8x^2 - 3x + 1$. Evaluate the function at $x = -2$.

4. An open-top box is formed by cutting squares out of a 5 inch by 7 inch piece of paper and then folding up the sides. The volume $V(x)$ in cubic inches of this type of open-top box is a function of the side length x in inches of the square cutouts and can be given by $V(x) = (7 - 2x)(5 - 2x)(x)$. Rewrite this equation by expanding the polynomial.

5. A rectangular playground space is to be fenced in using the wall of a daycare building for one side and 200 meters of fencing for the other three sides. The area $A(x)$ in square meters of the playground space is a function of the length x in meters of each of the sides perpendicular to the wall of the daycare building.

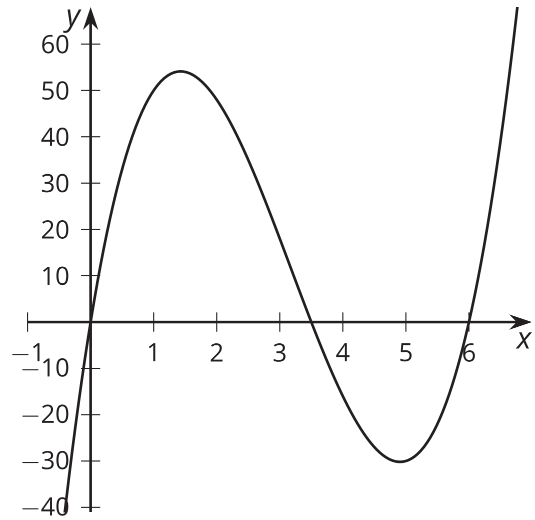
   a. What is the area of the playground when $x = 50$?

   b. Write an expression for $A(x)$.

   c. What is a reasonable domain for A in this context?

(From Unit 2, Lesson 1.)

6. Tyler finds an expression for $V(x)$ that gives the volume of an open-top box in cubic inches in terms of the length x in inches of the square cutouts used to make it. This is the graph Tyler gets if he allows x to take on any value between -1 and 7.



a. What would be a more appropriate domain for Tyler to use instead?

b. What is the approximate maximum volume for his box?

(From Unit 2, Lesson 1.)

Lesson 3: Introducing Polynomials

- Let's see what polynomials can look like.

3.1: Which One Doesn't Belong: What are Polynomials?

Which one doesn't belong?

A: $4 - x^2 + x^3 - 4x$

B: $2x^4 + x^2 - 5.7x + 2$

C: $x^2 + 7x - x^{\frac{1}{3}} + 2$

D: $x^5 + 8.36x^3 - 2.4x^2 + 0.32x$

3.2: Card Sort: Equations and Graphs

Your teacher will give you a set of cards. Group them into pairs that represent the same polynomial function. Be prepared to explain your reasoning.

3.3: Let's Make Some Curves

Use graphing technology to write equations for polynomial functions whose graphs have the characteristics listed when plotted on the coordinate plane.

1. A 1st degree polynomial function whose graph intercepts the vertical axis at 8.

2. A 2nd degree polynomial function whose graph has only positive y-values.

3. A 2nd degree polynomial function whose graph contains the point $(0, -9)$.

4. A 3rd degree polynomial function whose graph crosses the horizontal axis more than once.

5. A 4th degree or higher polynomial function whose graph never crosses the horizontal axis.

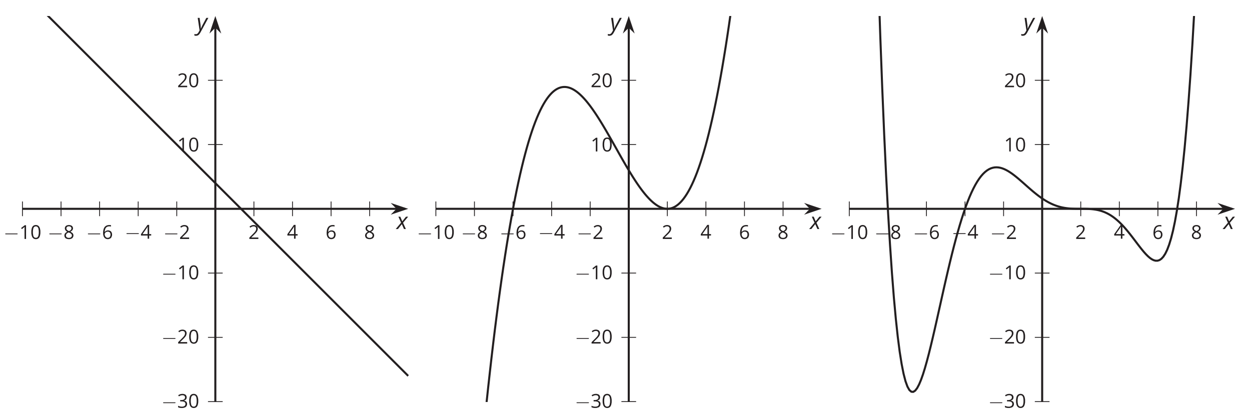
Are you ready for more?

For each of the following letters, find the equation for a polynomial function whose graph resembles the given letter: U, N, M, W.

Lesson 3 Summary

A polynomial function of x is a function given by a sum of terms, each of which is a constant times a whole number power of x. Polynomials are often classified by the term with the highest exponent on the independent variable. For example, a quadratic function, like $g(t) = 10 + 96t - 16t^2$, is considered a 2nd-**degree** polynomial because the highest exponent on t is 2. Similarly, a linear function like $f(x) = 3x - 10$ is considered a 1st-degree polynomial. Earlier, we considered the function $V(x) = (11 - 2x)(8.5 - 2x)(x)$, which gives the volume, in cubic inches, of a box made by removing the squares of side length x, in inches, from each corner of a rectangle of paper and then folding up the 4 sides. This is an example of a 3rd-degree polynomial, which is easier to see if we use the distributive property to rewrite the equation as $V(x) = 4x^3 - 39x^2 + 93.5x$.

Graphs of polynomials have a variety of appearances. Here are three graphs of different polynomials with degree 1, 3, and 6, respectively:



Since graphs of polynomials can curve up and down multiple times, they can have points that are higher or lower than the rest of the points around them. These points are **relative maximums** and **relative minimums**. In the second graph, there is a relative maximum at about $(-3, 18)$ and a relative minimum at $(2, 0)$. The word relative is used because while these are maximums and minimums relative to surrounding points, there are other points that are higher or lower.

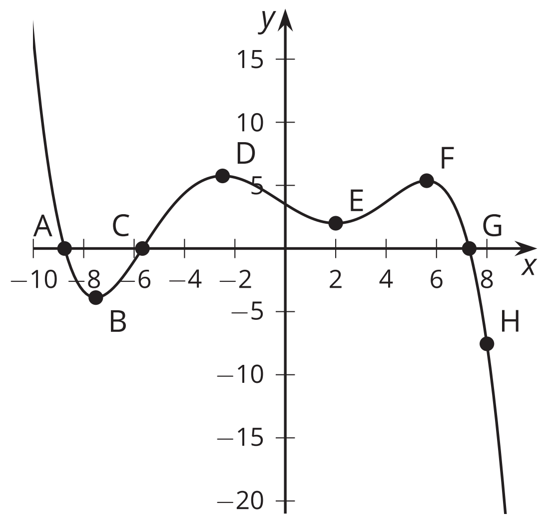
In future lessons, we'll explore connections between equations and graphs of polynomials and learn more about how the degree of a polynomial affects the shape of the graph.

Glossary

- degree
- relative maximum
- relative minimum

Lesson 3 Practice Problems

1. Select **all** points where relative minimum values occur on this graph of a polynomial function.



   A. Point A

   B. Point B

   C. Point C

   D. Point D

   E. Point E

   F. Point F

   G. Point G

   H. Point H

2. Add one term to the polynomial expression $14x^{19} - 9x^{15} + 11x^4 + 5x^2 + 3$ to make it into a 22nd degree polynomial.

3. Identify the degree, leading coefficient, and constant value of each of the following polynomials:

   a. $f(x) = x^3 - 8x^2 - x + 8$

   b. $h(x) = 2x^4 + x^3 - 3x^2 - x + 1$

   c. $g(x) = 13.2x^3 + 3x^4 - x - 4.4$

4. We want to make an open-top box by cutting out corners of a square piece of cardboard and folding up the sides. The cardboard is a 9 inch by 9 inch square. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts.

   a. Write an expression for $V(x)$.

   b. What is the volume of the box when $x = 1$?

   c. What is a reasonable domain for V in this context?

   (From Unit 2, Lesson 1.)

5. Consider the polynomial function p given by $p(x) = 7x^3 - 2x^2 + 3x + 10$. Evaluate the function at $x = \text{-}3$.

   (From Unit 2, Lesson 2.)

6. An open-top box is formed by cutting squares out of an 11 inch by 17 inch piece of paper and then folding up the sides. The volume $V(x)$ in cubic inches of this type of open-top box is a function of the side length x in inches of the square cutouts and can be given by $V(x) = (17 - 2x)(11 - 2x)(x)$. Rewrite this equation by expanding the polynomial.

   (From Unit 2, Lesson 2.)