Lesson 1: Let’s Make a Box

- Let’s investigate volumes of different boxes.

1.1: Which One Doesn’t Belong: Boxes

Which one doesn’t belong?

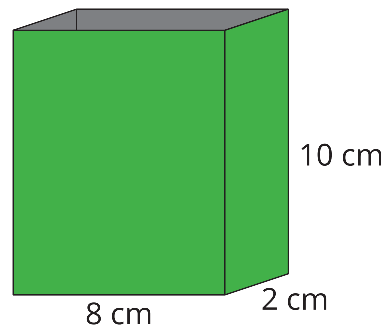
A.

length: 4 cm

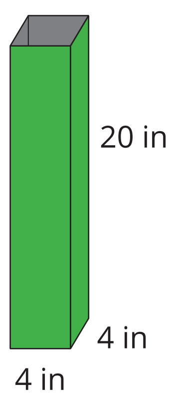
width: 8 cm

height: 10 cm

B.

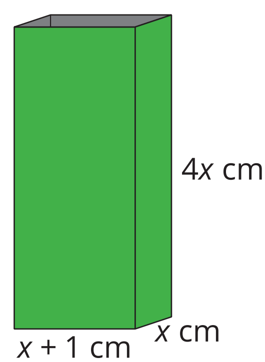


C.



D.

$V = 320$ cm^3



1.2: Building Boxes

Your teacher will give you some supplies.

1. Construct an open-top box from a sheet of paper by cutting out a square from each corner and then folding up the sides.

2. Calculate the volume of your box, and complete the table with your information.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
1				

1.3: Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

   Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

3. Use graphing technology to create a graph representing $V(x)$. Approximate the value of x that would allow you to construct an open-top box with the largest volume possible from one piece of paper.

Are you ready for more?

The surface area $A(x)$ in square inches of the open-top box is also a function of the side length x in inches of the square cutouts.

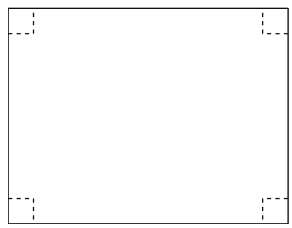
1. Find one expression for $A(x)$ by summing the area of the five faces of our open-top box.

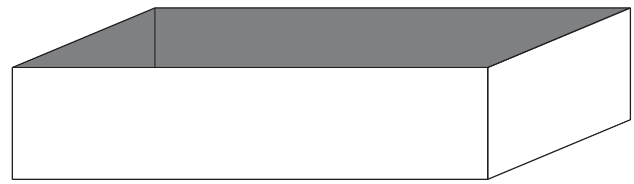
2. Find another expression for $A(x)$ by subtracting the area of the cutouts from the area of the paper.

3. Show algebraically that these two expressions are equivalent.

Lesson 1 Summary

Polynomials can be used to model lots of situations. One example is to model the volume of a box created by removing squares from each corner of a rectangle of paper.





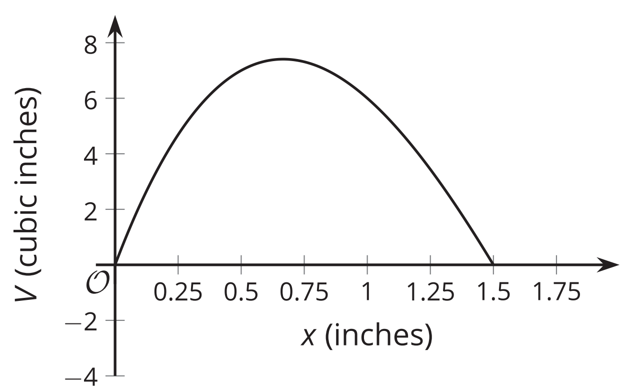
Let $V(x)$ be the volume of the box in cubic inches where x is the side length in inches of each square removed from the four corners.

To define V using an expression, we can use the fact that the volume of a cube is $(length)(width)(height)$. If the piece of paper we start with is 3 inches by 8 inches, then:

$$V(x) = (3 - 2x)(8 - 2x)(x)$$

What are some reasonable values for x? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since $3 - 1.5 \cdot 2 = 0$). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of $y = V(x)$. It looks like the largest volume we can get for a box made this way from a 3 inch by 8 inch piece of paper is about 7.4 in^3.



Glossary

- polynomial

Lesson 1 Practice Problems

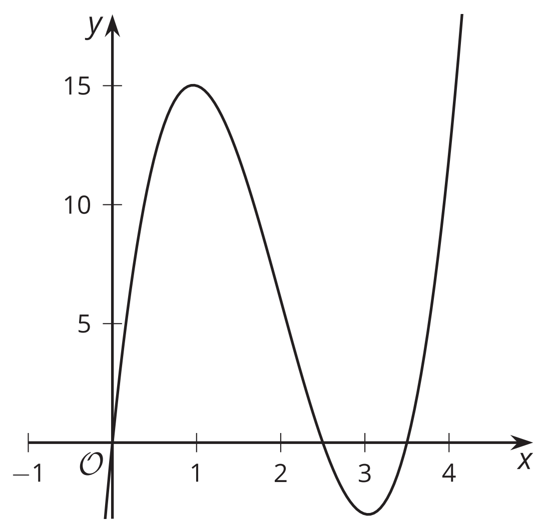
1. A rectangular schoolyard is to be fenced in using the wall of the school for one side and 150 meters of fencing for the other three sides. The area $A(x)$ in square meters of the schoolyard is a function of the length x in meters of each of the sides perpendicular to the school wall.

    a. Write an expression for $A(x)$.

    b. What is the area of the schoolyard when $x = 40$?

    c. What is a reasonable domain for A in this context?

2. Noah finds an expression for $V(x)$ that gives the volume of an open-top box in cubic inches in terms of the length x in inches of the cutout squares used to make it. This is the graph Noah gets if he allows x to take on any value between -1 and 5.



    a. What would be a more appropriate domain for Noah to use instead?

    b. What is the approximate maximum volume for his box?

3. Mai wants to make an open-top box by cutting out corners of a square piece of cardboard and folding up the sides. The cardboard is 10 centimeters by 10 centimeters. The volume $V(x)$ in cubic centimeters of the open-top box is a function of the side length x in centimeters of the square cutouts.

   a. Write an expression for $V(x)$.

   b. What is the volume of the box when $x = 3$?

4. The area of a pond covered by algae is $\frac{1}{4}$ of a square meter on day 1 and it doubles each day. Complete the table.

day	1	2	3	4	5	6
area of algae in square meters						

(From Unit 1, Lesson 2.)

5. Here is a table showing values of sequence p. Define p recursively using function notation.

n	$p(n)$
1	5,000
2	500
3	50
4	5
5	0.5

(From Unit 1, Lesson 6.)

6. The table shows two sloth populations growing over time.

time (years since 1990)	**population 1 (thousands)**	**population 2 (thousands)**
0	90.0	39
1	76.5	37
2	65.0	35
3	55.3	33
4	47.0	31
5		
6		
7		
8		

a. Describe a pattern in how each population changed from one year to the next.

b. These patterns continued for many years. Based on this information, fill in the extra rows in the table.

c. On the same axes, draw graphs of the two populations over time.

d. Does Population 2 ever equal Population 1? If so, when? Explain or show your reasoning.

(From Unit 1, Lesson 10.)