

Related Rates

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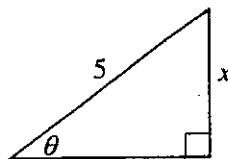
26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V ? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

93

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

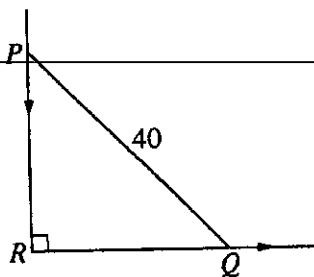
(A) $\frac{1}{\pi}$ (B) $\frac{1}{2}$ (C) $\frac{2}{\pi}$ (D) 1 (E) 2



97
BC

23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

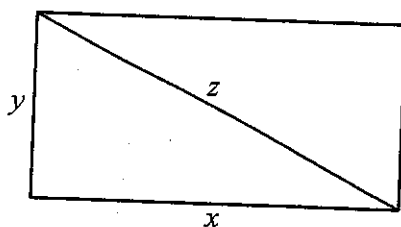
(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12



93
BC

34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32



- 88 40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3 \frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

- 88 BC 37. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$

- 88 BC 78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C , what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$
 (B) $-(0.1)C$
 (C) $-\frac{(0.1)C}{2\pi}$
 (D) $(0.1)^2 C$
 (E) $(0.1)^2 \pi C$

- 93 34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

- (A) $-\frac{7}{8}$ feet per minute
(B) $-\frac{7}{24}$ feet per minute
(C) $\frac{7}{24}$ feet per minute
(D) $\frac{7}{8}$ feet per minute
(E) $\frac{21}{25}$ feet per minute

- 64 9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

- (A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

- 85 31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

- (A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

- 85 22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

- (A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

- 98 90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

- 97 81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

- 97 86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

- 98 AC 20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k ?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12