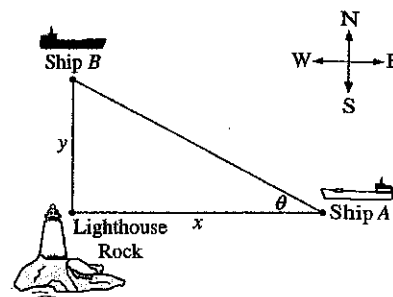


**AP<sup>®</sup> CALCULUS AB**  
**2002 SCORING GUIDELINES (Form B)**

**Question 6**

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let  $x$  be the distance between Ship A and Lighthouse Rock at time  $t$ , and let  $y$  be the distance between Ship B and Lighthouse Rock at time  $t$ , as shown in the figure above.



- Find the distance, in kilometers, between Ship A and Ship B when  $x = 4$  km and  $y = 3$  km.
- Find the rate of change, in km/hr, of the distance between the two ships when  $x = 4$  km and  $y = 3$  km.
- Let  $\theta$  be the angle shown in the figure. Find the rate of change of  $\theta$ , in radians per hour, when  $x = 4$  km and  $y = 3$  km.

(a) Distance =  $\sqrt{3^2 + 4^2} = 5$  km

(b)  $r^2 = x^2 + y^2$

$$2r \frac{dr}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

or explicitly:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dt} = \frac{1}{2\sqrt{x^2 + y^2}} \left( 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right)$$

At  $x = 4$ ,  $y = 3$ ,

$$\frac{dr}{dt} = \frac{4(-15) + 3(10)}{5} = -6 \text{ km/hr}$$

(c)  $\tan \theta = \frac{y}{x}$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{\frac{dy}{dt} x - \frac{dx}{dt} y}{x^2}$$

At  $x = 4$  and  $y = 3$ ,  $\sec \theta = \frac{5}{4}$

$$\frac{d\theta}{dt} = \frac{16}{25} \left( \frac{10(4) - (-15)(3)}{16} \right)$$

$$= \frac{85}{25} = \frac{17}{5} \text{ radians/hr}$$

1 : answer

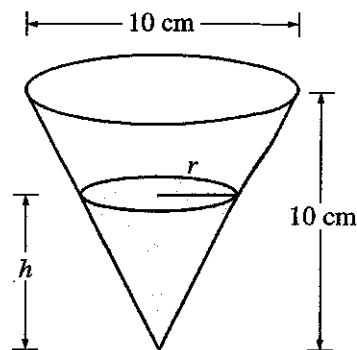
4 { 1 : expression for distance  
 2 : differentiation with respect to  $t$   
 < -2 > chain rule error  
 1 : evaluation

4 { 1 : expression for  $\theta$  in terms of  $x$  and  $y$   
 2 : differentiation with respect to  $t$   
 < -2 > chain rule, quotient rule, or  
 transcendental function error  
 note: 0/2 if no trig or inverse trig  
 function  
 1 : evaluation

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## Question 5

A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth  $h$  is changing at the constant rate of  $-\frac{3}{10}$  cm/hr.



(The volume of a cone of height  $h$  and radius  $r$  is given by  $V = \frac{1}{3}\pi r^2 h$ .)

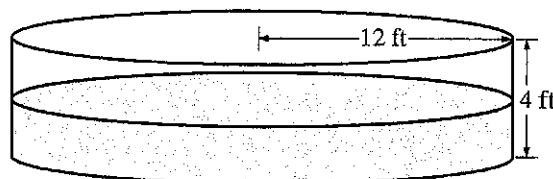
- Find the volume  $V$  of water in the container when  $h = 5$  cm. Indicate units of measure.
- Find the rate of change of the volume of water in the container, with respect to time, when  $h = 5$  cm. Indicate units of measure.
- Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

<p>(a) When <math>h = 5</math>, <math>r = \frac{5}{2}</math>; <math>V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi \text{ cm}^3</math></p> <p>(b) <math>\frac{r}{h} = \frac{5}{10}</math>, so <math>r = \frac{1}{2}h</math></p> $V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3; \frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$ $\left.\frac{dV}{dt}\right _{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$ <p style="text-align: center;">OR</p> $\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt}\right); \frac{dr}{dt} = \frac{1}{2} \frac{dh}{dt}$ $\left.\frac{dV}{dt}\right _{h=5, r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$	<p>1 : <math>V</math> when <math>h = 5</math></p> <p>1 : <math>r = \frac{1}{2}h</math> in (a) or (b)</p> <p>1 : <math>\left\{ \begin{array}{l} V \text{ as a function of one variable} \\ \text{in (a) or (b)} \end{array} \right.</math> OR</p> <p>5 : <math>\frac{dr}{dt}</math></p> <p>2 : <math>\frac{dV}{dt}</math></p> <p>&lt; -2 &gt; chain rule or product rule error</p>
$= -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$ <p>(c) <math>\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = -\frac{3}{40}\pi h^2</math></p> $= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10} \cdot \text{area}$ <p>The constant of proportionality is <math>-\frac{3}{10}</math>.</p>	<p>1 : evaluation at <math>h = 5</math></p> <p>1 : shows <math>\frac{dV}{dt} = k \cdot \text{area}</math></p> <p>2 : 1 : identifies constant of proportionality</p>
<p>units of <math>\text{cm}^3</math> in (a) and <math>\text{cm}^3/\text{hr}</math> in (b)</p>	<p>1 : correct units in (a) and (b)</p>

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**2010 SCORING GUIDELINES (Form B)**

**Question 3**

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, water is pumped into the pool at the rate  $P(t)$  cubic feet per hour. The table above gives values of  $P(t)$  for selected values of  $t$ . During the same time interval, water is leaking from the pool at the rate  $R(t)$  cubic feet per hour, where  $R(t) = 25e^{-0.05t}$ . (Note: The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)

- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval  $0 \leq t \leq 12$  hours. Show the computations that lead to your answer.
- Calculate the total amount of water that leaked out of the pool during the time interval  $0 \leq t \leq 12$  hours.
- Use the results from parts (a) and (b) to approximate the volume of water in the pool at time  $t = 12$  hours. Round your answer to the nearest cubic foot.
- Find the rate at which the volume of water in the pool is increasing at time  $t = 8$  hours. How fast is the water level in the pool rising at  $t = 8$  hours? Indicate units of measure in both answers.

(a)  $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 :  $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b)  $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 :  $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c)  $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time  $t = 12$  hours, the volume of water in the pool is approximately 1434  $\text{ft}^3$ .

(d)  $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left. \frac{dh}{dt} \right|_{t=8} = \frac{1}{144\pi} \cdot \left. \frac{dV}{dt} \right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

4 :  $\begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left. \frac{dh}{dt} \right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

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**2008 SCORING GUIDELINES**

**Question 3**

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume  $V$  of a right circular cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)

- (a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?
- (b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is  $R(t) = 400\sqrt{t}$  cubic centimeters per minute, where  $t$  is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time  $t$  when the oil slick reaches its maximum volume. Justify your answer.
- (c) By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part (b).

- (a) When  $r = 100$  cm and  $h = 0.5$  cm,  $\frac{dV}{dt} = 2000$  cm<sup>3</sup>/min  
 and  $\frac{dr}{dt} = 2.5$  cm/min.

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039 \text{ cm/min}$$

$$4 : \begin{cases} 1 : \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2 : \text{expression for } \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$$

- (b)  $\frac{dV}{dt} = 2000 - R(t)$ , so  $\frac{dV}{dt} = 0$  when  $R(t) = 2000$ .

This occurs when  $t = 25$  minutes.

Since  $\frac{dV}{dt} > 0$  for  $0 < t < 25$  and  $\frac{dV}{dt} < 0$  for  $t > 25$ ,

the oil slick reaches its maximum volume 25 minutes after the device begins working.

$$3 : \begin{cases} 1 : R(t) = 2000 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

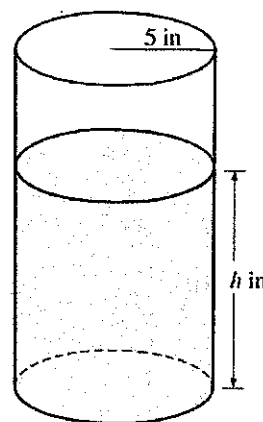
- (c) The volume of oil, in cm<sup>3</sup>, in the slick at time  $t = 25$  minutes is given by  $60,000 + \int_0^{25} (2000 - R(t)) dt$ .

$$2 : \begin{cases} 1 : \text{limits and initial condition} \\ 1 : \text{integrand} \end{cases}$$

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2003 SCORING GUIDELINES**

**Question 5**

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let  $h$  be the depth of the coffee in the pot, measured in inches, where  $h$  is a function of time  $t$ , measured in seconds. The volume  $V$  of coffee in the pot is changing at the rate of  $-5\pi\sqrt{h}$  cubic inches per second. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)



- (a) Show that  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ .
- (b) Given that  $h = 17$  at time  $t = 0$ , solve the differential equation  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$  for  $h$  as a function of  $t$ .
- (c) At what time  $t$  is the coffeepot empty?

(a)  $V = 25\pi h$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} = -5\pi\sqrt{h}$$

$$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$$

$$3 : \begin{cases} 1 : \frac{dV}{dt} = -5\pi\sqrt{h} \\ 1 : \text{computes } \frac{dV}{dt} \\ 1 : \text{shows result} \end{cases}$$

(b)  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$$\frac{1}{\sqrt{h}} dh = -\frac{1}{5} dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

$$2\sqrt{17} = 0 + C$$

$$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$$

$$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } h = 17 \\ \quad \text{when } t = 0 \\ 1 : \text{solves for } h \end{cases}$$

(c)  $\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$

$$t = 10\sqrt{17}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

1 : answer